DEVELOPMENT OF A VISCOELASTIC CONTINUUM DAMAGE MODEL FOR CYCLIC LOADING

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Abstract

A previously developed spectrum model for linear viscoelastic behavior of solids is used to describe the rate-dependent damage growth of a time dependent material under cyclic loading. Through the use of the iterative solution of a special Volterra integral equation, the cyclic strain history is described. The spectrum-based model is generalized for any strain rate and any uniaxial load history to formulate the damage function. Damage evolution in the body is described through the use of a rate-type evolution law which uses a pseudo strain to express the viscoelastic constitutive equation with damage. The resulting damage function is used to formulate a residual strength model. The methodology presented is demonstrated by comparing the peak values of the computed cyclic strain history as well as the residual strength model predictions to the experimental data of a polymer matrix composite.

Keywords: creep, fatigue, damage mechanics, constitutive model, Volterra integral equation.

1. Introduction

Materials such as fiber reinforced polymer composites exhibit time dependent behavior, due primarily to the matrix component. The viscoelastic response from the polymer component can become accelerated due to a combination of both creep and cyclic loading which the material experiences in cases like tension-tension fatigue [Vinogradov 2001]. Even at low loads (well below the ultimate tensile stress) damage in the form of matrix cracks and fiber breaks occurs at a relatively low number of cycles [Talreja 1987; Harris 2003; Case and Reifsnider 2003; Reifsnider 1991]. Therefore, it is important to develop a damage model that also includes the rate dependent nature of the viscoelastic material subjected to cyclic loading.

Damage modeling of composites is not an easy task, given the evolution of various damage mechanisms (matrix cracking, fiber breakage, interfacial debonding, transverse-ply cracking, and ply delamination) in composite materials [Talreja 1987]. One of the approaches used to deal with such diverse damage states is to use a continuous damage variable, which usually relies on the concept of effective stress and strain equivalence [Lemaitre 1984; Lemaitre and Dufailly 1987; Kachanov 1986; Jessen and Plumtree 1991]. The effective stress theory assumes that the damage is uniformly distributed, which in fiber reinforced composites, the complex damage occurs in a distributed manner with the damage mechanisms occurring in diffused areas. The effective stress theory is used in this study to introduce the damage variable S which is defined to be $S = 0$ for an undamaged material; as damage progresses, the load bearing area decreases and the net stress approaches infinity with the damage variable approaching its largest value ($S=1$) [Lemaitre 1984; Kachanov 1986; Stigh 2006].

Having established the concept of the damage parameter, the rate dependent nature of the material must be integrated into the model. The effects of both viscoelasticity and damage have been addressed in several studies [Schapery 1980a, 1980b, 1990, 1994; Weitsman 1988, Kim and Little 1990; Park, et.al. 1996; Schapery and Sicking 1995]. To develop a damage model, Schapery [Schapery 1980a, 1984] formulated the modified elastic-viscoelastic correspondence principle, which is applicable for both linear and nonlinear materials. Using this formulation, he was able to express the viscoelastic constitutive equation in a form similar to that of an elastic equation through the use of pseudo variables. Schapery's approach is used in this paper to represent the viscoelastic nature of the composite through the use of a pseudo strain, which contains the effect of the convolution integral (hereditary effects). The damage state is then described by a continuous damage variable that uses the pseudo strain response of the composite laminate.

The primary objective of this study is to establish an alternative methodology in the study of viscoelastic behavior which can be used in the prediction of degrading properties and residual strength. A constitutive model for isotropic homogenous viscoelastic solids using a distribution function developed earlier [Sullivan 2006] is first reviewed and then extended to include cyclic loading. The constitutive model as a function of the pseudo strain is then used to form the continuum mechanics damage parameter (S) from which a residual strength model is proposed. Experimental details consisting of the cyclic response and residual strength are given for a quasi-isotropic polymer matrix composite material. As an initial demonstration of the methodology, the composite laminate is regarded as an effective homogenous continuum and the predictions are compared to the experimental data.

2. Spectrum-based viscoelastic model for creep and fatigue

A previously developed spectrum model [Sullivan 2006] for viscoelastic materials has been extended to include fatigue loading so that the effect of fatigue – creep interaction can be included. For the sake of completeness, a brief overview of the continuous spectrum model is given.

Beginning from the Boltzman-Volterra theory, the constitutive equation describing a linear isotropic viscoelastic solid without damage in Cartesian coordinates is given as

$$T_{kl} = \delta_{kl} \int_{-\infty}^{t} [\lambda_e + \lambda_v (t-\tau)] \frac{\partial \varepsilon_{rr}}{\partial \tau} d\tau + 2 \int_{-\infty}^{t} [\mu_e + \mu_v (t-\tau)] \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau \tag{1}$$

where a repeated index implies a sum and ε_{rr} is the trace of the strain tensor **ε.** The parameters λ_e and μ_e are the Lame' elastic constants and $\lambda_v(t)$ and $\mu_v(t)$ are the viscoelastic functions of the material. Since all properties of importance can be expressed through the use of the Lame´ parameters, the Lame´ functions $\lambda_v(t)$ and $\mu_v(t)$ must be determined. A form of the Lame´ functions is formulated as [Eringen 1967],

$$\lambda_v(t) = -\lambda_e \int_0^\infty \phi_1(\alpha) \left[1 - e^{\frac{-t}{\alpha}} \right] d\alpha \tag{2a}$$

$$\mu_v(t) = -\mu_e \int_0^\infty \phi_2(\alpha) \left[1 - e^{\frac{-t}{\alpha}} \right] d\alpha \tag{2b}$$

where $\varphi(\alpha)$, called the relaxation spectrum, satisfies two criteria:

$$\phi(\alpha) \geq 0 \quad \phi(\infty) \leq 1 \tag{3}$$

and for ease of computation and demonstration in this study, only one spectrum function has been taken, i.e., $\phi_1(\alpha) = \phi_2(\alpha) = \phi(\alpha)$. The choice of the distribution function is made when two additional criteria are imposed. First, the function must have a monotonic behavior and secondly it must reduce, in a limiting procedure to the delta function, giving rise to the constants appearing in both Kelvin-Voigt and non Kelvin-Voigt materials. Satisfying all the imposed criteria, the resulting function used for this study is

$$\phi(\alpha) = \frac{n}{\pi\left(1 + n_0^2 \alpha^2\right)} \tag{4}$$

with materials constants n and n_o having dimension of [1/sec]. In the distribution function selection process, the behavior of the chosen spectrum function $\varphi(\alpha)$ has been compared to the behavior of other distributions such as the Poisson, Maxwell or the Chi functions. It is noted that when compared to the latter three, $\varphi(\alpha)$ can be used for all times [$-\infty$, $+\infty$] and it also most closely mimics the behavior of most viscoelastic materials because its response closely follows a normal probability distribution function.

Having selected the spectrum function $\varphi(\alpha)$, the Lame´ parameters $\lambda_v(t)$ and $\mu_v(t)$ can be determined and all properties and responses can be expressed for viscoelastic materials. The full development of the time dependent response and properties using the given spectrum function can be found in Ref. [21]. The resulting strain as a function of time is determined to be

$$\varepsilon_{11}(t) = \frac{\sigma(t)}{E_e} + \frac{n}{\pi} \int_0^t \varepsilon_{11}(t-\tau)\, \psi(\tau) d\tau \tag{5}$$

where

$$\psi(t) = \int_{n_o t}^{\infty} \frac{\cos(z - n_o t)}{z} dz \tag{6a}$$

and

$$\sigma(t) = \begin{cases} \sigma_o, & \text{uniaxial creep} \\ \sigma_m (1 + A \sin(\omega t)), & \text{cyclic loading} \end{cases} \tag{6b}$$

with the mean strain σ_m, the amplitude A, $\omega = 2\pi f$, and f being the frequency of the test in cycles per second. Equation (5) is a Volterra integral equation and it is solved by the method of iteration, which will be discussed in Section 7. Having the expressions for the Lame´ functions $\lambda_v(t)$ and $\mu_v(t)$, the time dependent tensile modulus is obtained as

$$E(t) = E_e\left[\left(1-\frac{n}{2n_o}\right)+\frac{n}{\pi n_o}\varphi(n_o t)\right] \tag{7a}$$

where

$$\varphi(n_o t) = \int_{n_o t}^{\infty} \frac{\sin(z-n_o t)}{z}\, dz \tag{7b}$$

and E_e is the initial elastic modulus of the material. In this paper, the viscoelastic parameters n and n_o are determined by using the strain data at the superimposed stress and using Eq. (5) for the constant stress case for which $\sigma(t) = \sigma_{mean} = \sigma_o$. Once these parameters have been computed, the strain response for the cyclic loading $\sigma(t) = \sigma_m\{1 + A\sin(\omega t)\}$ is generated. This is the first validation step as the peak values of the computed response are compared to the measured peak strain for a fatigue test (see Fig. 4). Using this technique, complete analytical expressions for the strain response along with properties such as the modulus are obtained.

3. The Constitutive Damage Model Using Pseudo Strain

Attention is now turned to developing a methodology to incorporate damage into the linear viscoelastic model. Taking guidance from previous work [Park et.al. 1996; Schapery 1984], an alternate way of expressing the time dependency of viscoelastic materials is through the use of pseudo variables. Originally proposed by Schapery, the methodology involves the use of pseudo parameters which do not necessarily represent physical quantities such as strains and stresses, but which are useful in transforming the viscoelastic stress-strain relationships into elastic-like equations [Schapery 1984, 1990]. The concept of pseudo strain is developed through the equation

$$\varepsilon^R(t) = \frac{1}{E_R}\int_0^t E(t-\tau)\,\frac{d\varepsilon_{11}}{d\tau}d\tau \tag{8}$$

where

$$\varepsilon^R(t) = \text{pseudo strain} \qquad E(t) = \text{modulus}$$
$$\varepsilon_{11}(t) = \text{actual strain} \qquad E_R = \text{reference modulus}$$

Once the pseudo strain has been defined by Eq. (8), the constitutive law for a viscoelastic material can be expressed in a form similar to Hooke's law of linear elasticity for all times as

$$\sigma(t) = E_R\, \varepsilon^R(t)\,. \tag{9}$$

The simplicity of this description is the integral form of the constitutive equation has now been absorbed by the pseudo strain which contains the memory aspect of the material. The constitutive law for damage using the pseudo strain can also be obtained by considering the simplest form of the free energy of deformation (Helmholtz). The pseudo strain energy density function W^R can be defined as a function of the pseudo strain and a damage variable S as

$$W^R = \frac{1}{2} M(S)\left(\varepsilon^R\right)^2 \tag{10}$$

where M(S) is a constitutive function of damage. The components of the stress may be obtained by taking the first derivative of W^R with respect to the pseudo strain, thereby obtaining

$$\sigma = \frac{\partial W^R}{\partial \varepsilon^R} = M(S)\, \varepsilon^R\,. \tag{11}$$

The constitutive function M(S) is also a function of time since the damage parameter S is a function of time, i.e., $M(S) = M[S(t)] = M(t)$. As a first attempt, using the relaxation modulus (for the stiffness function M(t)) as defined by Eq. (7)) in Eq. (8), the pseudo strain can be written as

$$\varepsilon^R(t) = \frac{E_e}{E_R}\left[\left(1-\frac{m}{2}\right)\left\{\varepsilon_{11}(t)-\varepsilon_{11}(0)\right\} + \frac{m}{\pi}\int_0^t \left\{n_o(t-\tau)\right\}\frac{d\varepsilon_{11}}{d\tau}\,d\tau\right]. \tag{12}$$

It can be seen from Eq. (12) that the time dependent strain response must be known to formulate the pseudo strain, which is a necessary parameter in the development of the damage model (Section 4). The behavior of the pseudo strain is shown in Fig. 4.

4. The Damage Law

A continuum damage mechanics approach is used to formulate the damage function. In this paper, the damage parameter S is defined through the use of an effective stress or net stress σ' [Kachanov 1986; Stigh 2006] as

$$\sigma' = \frac{\sigma}{1-S} \tag{13}$$

where σ is the applied stress. Essentially, the net stress is the true stress because the growth of damage decreases the load bearing area and the net stress is based on this reduced area or "net area". It can be seen from Eq. (13) that for an undamaged material, S = 0, and the effective stress equals the applied stress. As S approaches 1, the load bearing area decreases and the effective stress tends to infinity. To describe the development of damage, Kachanov's damage growth law [Kachanov 1958] is used as

$$\frac{dS}{dt} = C\left(\frac{\sigma(t)}{1-S}\right)^{\eta} \tag{14}$$

where C and η are material constants to be determined. From Eq. (14), it is observed that the damage growth will always be positive for a tensile-tensile fatigue loading, as is considered in this study. This damage description assumes uniform damage and uniform damage growth. Upon integration, the damage function becomes

$$S(t) = 1 - \left\{1 - C(\eta+1)\int_0^t \left[\sigma(t)\right]^{\eta} dt\right\}^{\frac{1}{\eta+1}}. \tag{15}$$

The rupture time t_{RS} can be determined by evaluating Eq. (15) for the case of constant loading, i.e., creep [Stigh 2006], under the condition that at failure S(t) =1, as given by

$$t_{RS} = \frac{1}{C(\eta+1)[\sigma_o]^{\eta}} \quad . \tag{16}$$

Using this equation, the constants C and η can be determined from creep-rupture test data. Solving Eq. (15) for the term $C(\eta+1)$, and using the definition of stress from Eq. (11), the damage function is now expressed in terms of the pseudo strain as

$$S(t) = 1 - \left\{ 1 - \frac{\int_0^t \left[M(t)\,\varepsilon^R(t) \right]^{\eta} dt}{\int_0^{t_R} \left[M(t)\,\varepsilon^R(t) \right]^{\eta} dt} \right\}^{\frac{1}{\eta+1}} \tag{17}$$

Correlated with a residual strength model, the damage function in Eq. (17) is useful in establishing the level of damage in a material due to cyclic loading. The behavior of this function is shown in Fig. 5a.

5. The Residual Strength Model

It is observed that the damage function inherently contains information about the degradation of strength in the material and its composition is seen to contain a “pseudo stress”, i.e., $M(t)\varepsilon^R(t)$. Taking guidance from the results of the damage function in Eq. (17), a form for the remaining or residual strength is proposed here by forming a factor Fr(t) as given by

$$Fr(t) = \left\{ 1 - \frac{\beta \int_0^t \left| M(t)\,\varepsilon^R(t) \right|^{\eta} dt}{\int_0^{t_f} \left| M(t)\,\varepsilon^R(t) \right|^{\eta} dt} \right\}^{\frac{1}{\eta+1}} \tag{18}$$

for a time in the range $[0,t_f]$ where t_f is some final time (not necessarily the rupture time) and the parameter β is to be determined by comparison with either the experimental or the empirical data. This form is similar to empirical residual strength models based on statistical distributions.

It is seen that when η approaches infinity in Eq. (18), the value of the residual strength factor Fr is unity, indicating an elastic material, i.e., no damage. Since the proposed residual strength function given in Eq. (18) is normalized with respect to the initial strength of the material X_T, the residual strength $\sigma_R(t)$ is formulated as

$$\sigma_R(t) = X_T \; Fr(t) \,. \tag{19}$$

A flow chart depicting the methodology is shown in Figure 1.

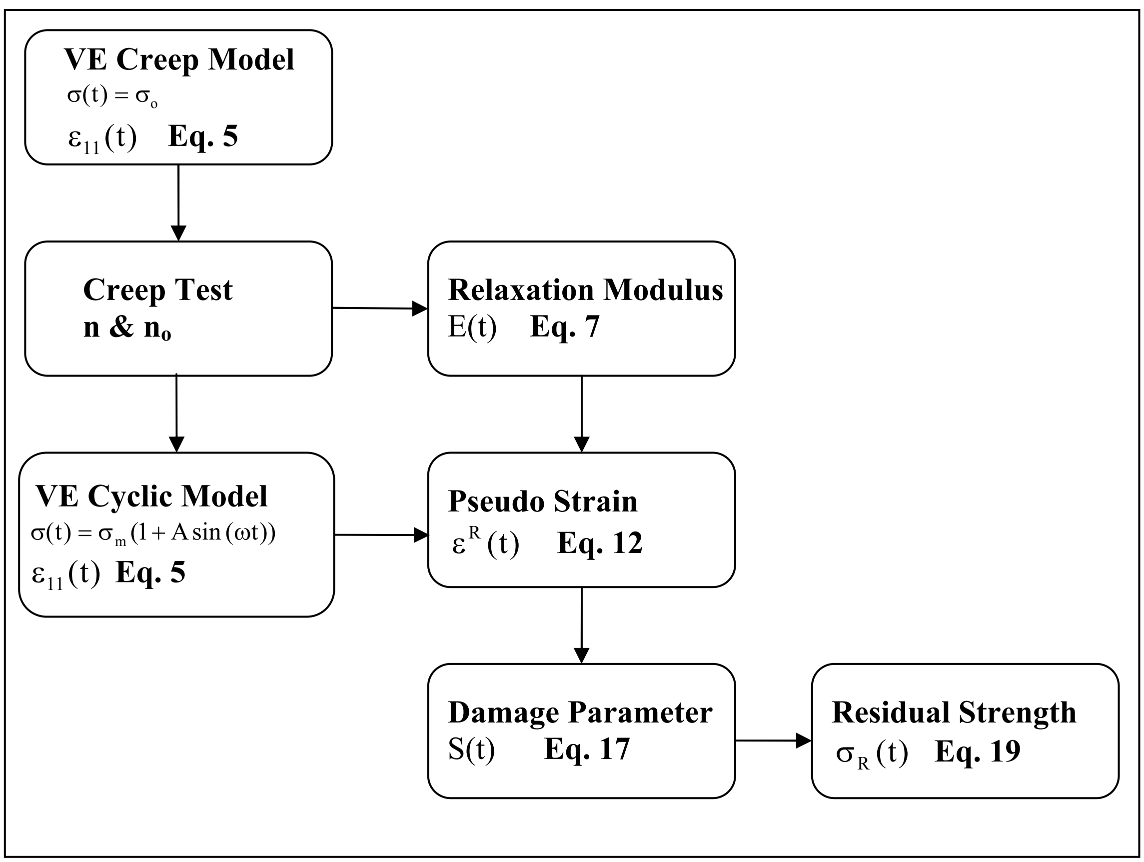


Fig. 1. Flow Chart for Viscoelastic Continuum Damage Mechanics Modeling.

6. Experimental Details

Material Description

The composite material, for which both fatigue to failure and residual strength data were available, is an E-glass woven roving in a vinyl ester resin manufactured by the Vacuum Assisted Resin Transfer Molding (VARTM) process. Details regarding the material system are given in Table 1. The quasi-isotropic ply orientation of the composite laminate is denoted by the warp fiber direction.

Table 1. Material System Details

Fiber system	Vetrotex 324 woven roving E-glass
Resin system	Ashland Derakane 8084 vinyl ester
Stacking sequence	$[0/+45/90/-45/0]_s$
Coupon size	6 X 25.4 X 150 mm
Tensile Strength (X_T) (ksi)	50.3
Tensile Modulus (msi)	3.3

Cyclic Testing

Strain data from constant amplitude cyclic testing at 5 Hz and stress ratio (minimum stress/ maximum stress) of R = 0.1 at three peak stress values (30 ksi, 22 ksi, and 16.5 ksi) is used to demonstrate the procedure previously described. All loading was performed by applying the load in the principal fiber direction (0°). Peak strain data (strain at the peak stress) and minimum strain (strain at the minimum stress) at each cycle is obtained. Typical response from the laminates at the three stress levels is shown in Fig. 2. Strain at the mean stress level (dotted line) is computed by averaging the peak and minimum strain values and henceforth used as the response of the material at the superimposed “creep” stress. The viscoelastic parameters that are to be extracted are based on the mean strain.

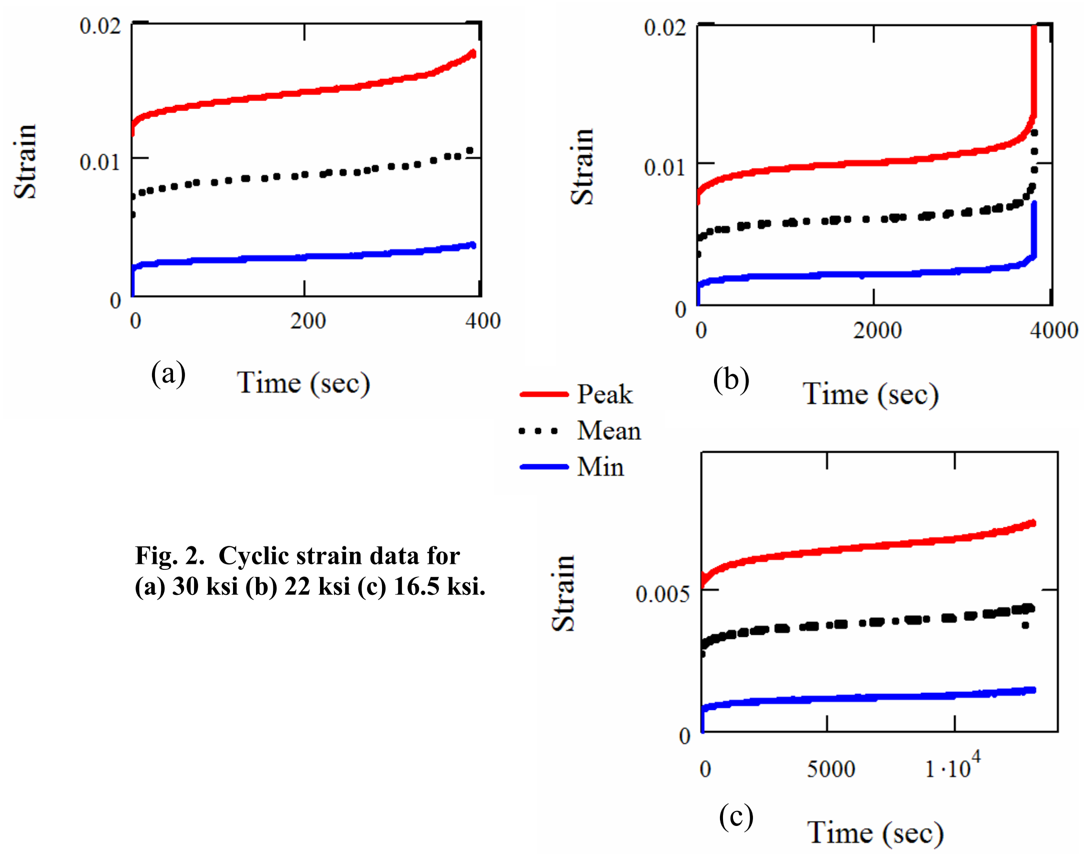


Fig. 2. Cyclic strain data for (a) 30 ksi (b) 22 ksi (c) 16.5 ksi.

7. Discussion of Results

As an initial demonstration of the methodology, the composite laminate is regarded as an effective homogenous continuum due to its quasi-isotropic lay-up sequence (c.f. Table 1). Additionally, all experimental data is obtained from uniaxial loads applied in-plane along the warp fiber direction (0°).

Determination of the viscoelastic parameters n_o and n

The two viscoelastic material parameters n_o and *n* are first determined from the mean strain data that is regarded as the superimposed experimental creep data. Knowing the strain values at the initial (t_i) , intermediate (t_m), and the final time (t_f), a transcendental equation for n_o is formed which is based on a two-term approximation of Eq. (5) as

$$C_1 + C_2 e^{-n_o t} = \varepsilon(t)$$

where

$$C_1 = \frac{\varepsilon_f - \frac{\sigma_o}{E_e} e^{-n_o t_f}}{1 - e^{-n_o t_f}}, \qquad C_2 = \varepsilon_i - C_1$$

and

ε_i = initial strain at time $t = t_i = 0$
ε_f = final strain at time $t = t_f$

Several strain values at intermediate times are tried and the value that best fits the experimental data is retained. Strain values, for the three peak stress levels demonstrated here, which are used to compute n_o and n are listed in Table 2.

Table 2. Viscoelastic modeling parameters.

Cyclic Peak Stress (ksi)	**30**	**22**	**16.5**
Normalized strength (Fa)	0.595	0.436	0.328
Initial strain (ε_i)	7.327×10^{-3}	4.378×10^{-3}	2.983×10^{-3}
Intermediate strain (ε_m)	8.381×10^{-3}	5.855×10^{-3}	3.788×10^{-3}
Final strain (ε_f)	9.579×10^{-3}	6.705×10^{-3}	4.062×10^{-3}
n_o	4.316×10^{-3}	9.196×10^{-4}	2.161×10^{-4}
$m = n/n_o$	0.642	0.839	0.675

Once n_o is determined, the viscoelastic parameter n is obtained by evaluating Eq. (5) at $t = t_f$ as

$$n = \frac{\pi\left(\varepsilon_{11}(t_f) - \sigma(t_f)/E_e\right)}{\int_0^{t_f} \varepsilon_{11}(t_f - \tau)\, \psi(\tau)\, d\tau}. \qquad (20)$$

This procedure is followed for each mean strain or "creep" response and the viscoelastic parameters n and n_o are determined for each stress value. The parameters are then substituted into Eq. (5) to obtain the analytical "creep" strain history which is compared to the experimental data shown in Fig. 3 on a strain vs. log(time) scale. Also shown in Fig. 3 and in Table 2 is the ratio m of the viscoelastic parameters given as

$$m = \frac{n}{n_o} \qquad (21)$$

for each stress level. The value of *m* does not change significantly for each stress regardless of the strain response used (peak, mean or minimum) because *m* is a *shape* parameter of the response curve, thereby representing the time-dependent characteristics of the material. It is also noted that *m* is a very weak function of the stress as it does not change significantly between each stress level. Once the viscoelastic parameters n_o and n have been determined, the cyclic strain response and the modulus E(t) are computed.

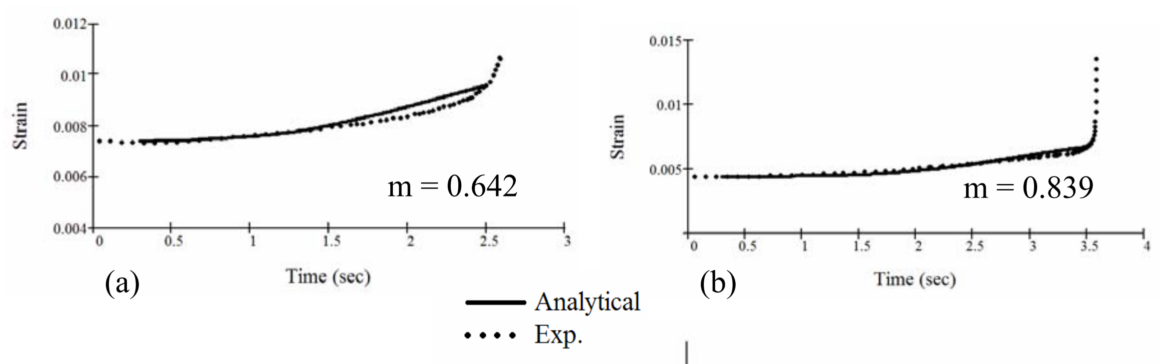
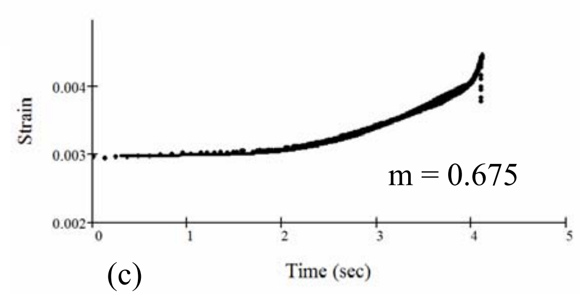


Fig. 3. Mean strain from "superimposed" stress regarded as "creep" strain for peak stress levels of (a) 30 ksi (b) 22 ksi (c) 16.5 ksi.



Iterative Solution of the Integral Equation

To begin the iterative process for determining the peak cyclic response, the strain is first normalized and the time is non-dimensionalized by using

$$\bar{\varepsilon}_{11}(t) = \frac{\varepsilon_{11}(t)}{\varepsilon_{11}(t_f)}, \qquad \lambda = n_o t \qquad .$$

Equations (5) and (6) are now written as

$$\bar{\varepsilon}_{11}(\lambda) = p(\lambda)\,\bar{\varepsilon}_{11}(0) + \frac{m}{\pi}\int_0^{\lambda} \bar{\varepsilon}_{11}(\lambda-\xi)\,\psi(\xi)\,d\xi \tag{22a}$$

where

$$m = n/n_0\ ,\ \ p(\lambda) = \sigma(\lambda)/\sigma_0\ ,\ \ \bar{\varepsilon}_{11}(0) = \sigma_0/E_e$$

$$\psi(\xi) = \int_{\xi}^{\infty} \frac{\cos(z-\xi)}{z}\,dz \tag{22b}$$

and

$$p(\lambda) = \begin{cases} 1 \ \text{ for uniaxial creep} \\ 1 + A\sin(\omega\lambda/n_o) \ \text{ for cyclic loading} \end{cases}. \tag{22c}$$

The integral equation (22a) is solved by iteration in which the zeroth approximation is developed by introducing a subsidiary solution

$$\varepsilon^{*}_{11}(\lambda) = \alpha_0 - \alpha_1 e^{-\lambda}$$

to be used in the convolution term of Eq. (22a), thereby obtaining

$$\left[\bar{\varepsilon}_{11}(\lambda)\right]^{(0)} = p(\lambda)\,\bar{\varepsilon}_{11}(0) + \frac{m}{\pi}\int_0^{\lambda} (\alpha_0 - \alpha_1 e^{-(\lambda-\xi)})\,\psi(\xi)\,d\xi .$$

Forming the functions

$$I_1 = \int_0^{\lambda} \psi(\xi)\,d\xi$$

$$I_2 = e^{-\lambda}\int_0^{\lambda} e^{\xi}\psi(\xi)\,d\xi$$

and then expressing the zeroth solution as

$$\left[\bar{\varepsilon}_{11}(\lambda)\right]^{(0)} = p(\lambda)\,\bar{\varepsilon}_{11}(0) + \frac{m}{\pi}\left[\alpha_0\, I_1(\lambda) - \alpha_1\, I_2(\lambda)\right]. \tag{23}$$

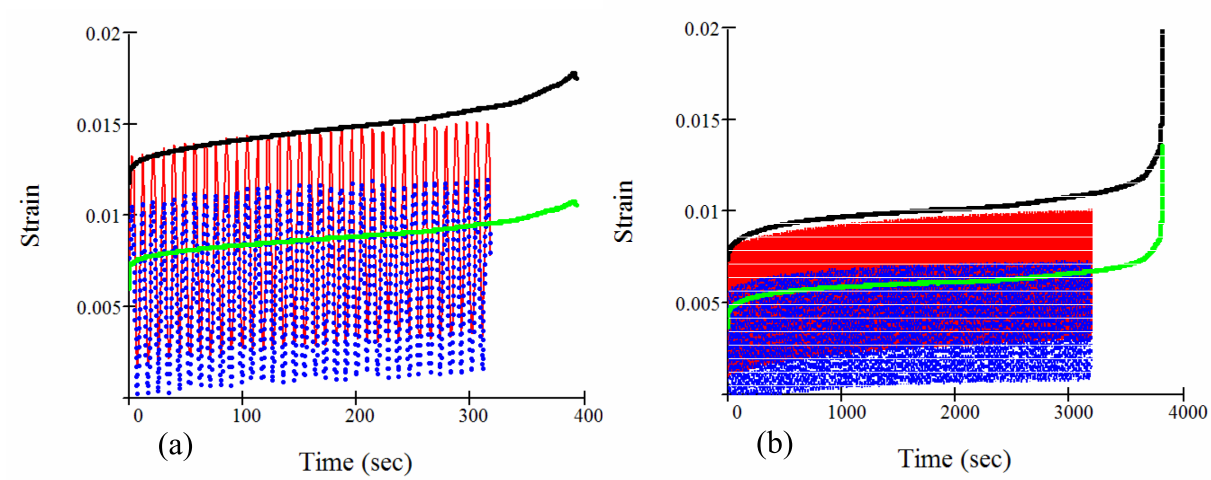
The coefficients α_0 and α_1 are determined by imposing the known values of $\bar{\varepsilon}_{11}$ at $\lambda = 0$ and at $\lambda = \lambda_f$. It is emphasized here that the previous procedure is used solely for the purpose of obtaining the zeroth approximation. Therefore, the values of α_0 and α_1 are used only in the zeroth approximation and are not material parameters. Using the solution of Eq. (23) as the zeroth solution in Eq. (22a), the successive solutions are obtained by iteration to convergence.

Using the above methodology, the cyclic strain history is computed to the selected final time for each stress level as shown in Fig. 4. The inset figure in Fig. 4c is shown to demonstrate the sinusoidal response. As can be seen, good correlation is obtained between the analytically obtained values and the measured peak strains until the experimental data shows a marked increase for all stress levels.

Having generated the cyclic strain, the strain rate can now be computed and the pseudo strain can be determined from Eq. (12). Fig. 4 also compares the behavior of the analytical cyclic strain and the pseudo strain. It is expected for the pseudo strain to be lower than the actual cyclic strain because the modulus (E(t)) in the definition of the pseudo strain (Eq. 8), is decreasing with increasing cycles.

Once the pseudo strain is obtained, the damage parameter S(t) is computed from Eq. (17) . It is noted that the spectrum based modulus E(t) is obtained from Eq. (7) which uses the viscoelastic parameters that effectively capture the peak response as shown in Fig. 4. It is reasonable to conclude that the peak response contains the degradation of the material. Therefore, E(t) is taken to describe the modulus degradation in the material and as a first approximation used to represent the constitutive damage function M(t).

The damage parameter S(t) is computed and shown in Fig. (5a) and its computation enables the correlation of various properties as functions of damage; as a way of quantifying the stiffness for a given level of damage, the relaxation modulus computed from Eq. (7) is correlated with the time dependent damage factor S and shown in Fig. 5b for the peak stress of 22 ksi.



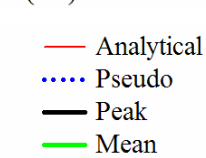
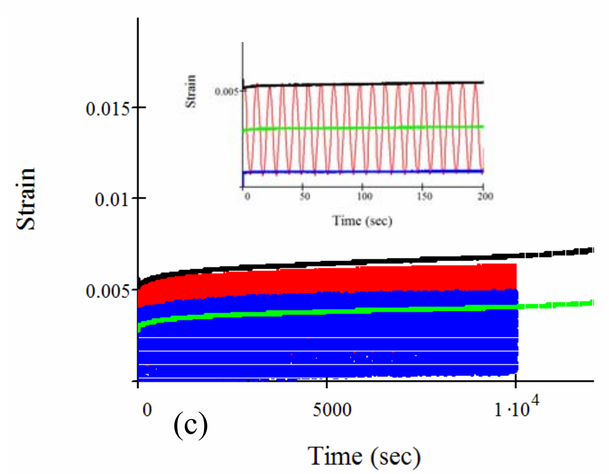


Fig. 4. Comparison of peak experimental, analytical, and pseudo strains for (a) 30 ksi (b) 22 ksi (c) 16.5 ksi.



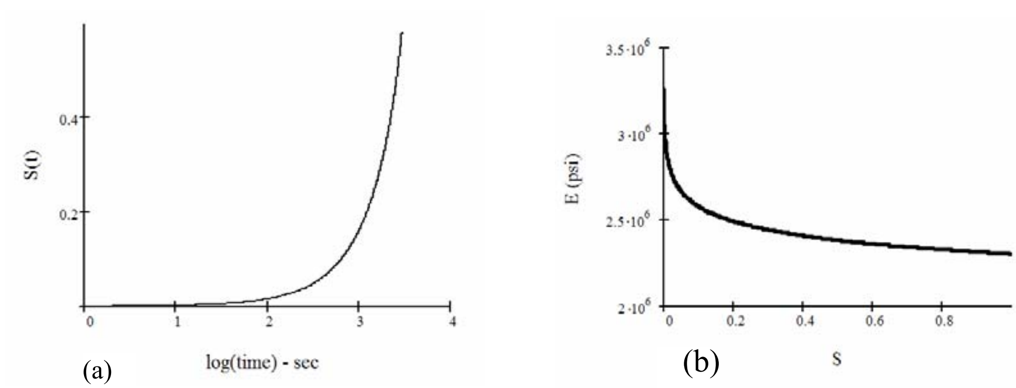


Fig. 5. Damage (a) S(t) (b) Modulus as a function of damage for peak stress of 22 ksi.

Finally, the residual strength is computed using Eq. (19) and Fig. 6 shows the residual strength curves normalized with respect to the initial strength (X_T) along with the experimental data for the 22 ksi (Fa = 0.436) and 16.5 ksi (Fa = 0.328) stress amplitudes; experimental residual strength data for the 30 ksi (Fa = 0.595) stress amplitude was not available. The model behavior compares favorably with phenomenological models such as those developed by Case and Reifsneider [2003]. Overall, good correlation is obtained between the experimental data and the residual strength model.

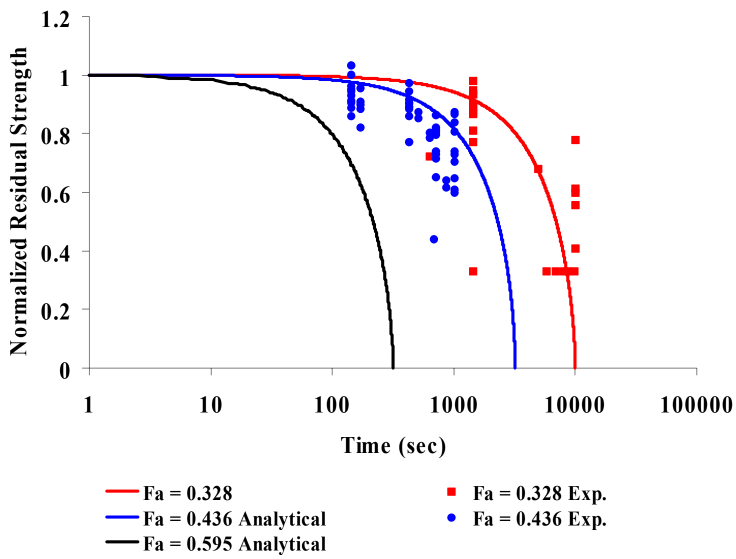


Fig. 6. Normalized residual strength for Fa = 0.328 (σ_{peak}=16.5 ksi), Fa = 0.436 (σ_{peak}= 22 ksi), Fa = 0.595 (σ_{peak}=30 ksi).

Using the technique discussed above, the peak cyclic response and the residual strength of a PMC material are predicted. This prediction is based on the mean strain or "creep" strain data of a single cyclic test, as explained earlier.

8. Conclusions

From an earlier development [Sullivan, 2006], a model to describe the creep response is modified to simulate the strain response of a viscoelastic material under cyclic loading. The resulting response is used to calculate the pseudo strain that is used to develop a damage model. Based on the results of the damage model, a residual strength function is formulated and compared to the experimental data. The model's behavior is similar to those of phenomenological models.

The methodology proposed in this study demonstrates a way to incorporate the material behavior of viscoelastic solids and damage growth due to superimposed static and cyclic loads. For each peak stress level, the model uses the mean response from a single constant amplitude fatigue test to predict the response and the residual strength. Due to its availability, experimental fatigue data for a quasi-isotropic polymer matrix composite laminate, tested in the principal material direction, is used for comparison with the analytical solutions. Although the composite was regarded as a continuous medium, acceptable agreement was obtained between the model predictions and the measured data, due mainly to its quasi-isotropic construction and its in-plane loading.

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REFERENCES

1. Vinogradov, A.M., Jenkins, C.H.M., Winter, R.M.: Cyclic loading effects on durability of polymer systems. In: Monteiro, P.J.M., Chong, K.P., Larsen-Basse, J., Komvopoulos, K. (eds.) Long Term Durability of Structural Materials, pp. 159-170. Elsevier (2001).

2. Talreja, R.: Fatigue of Composite Materials. Technomic Publishing Co., Lancaster, PA, (1987).

3. Harris, B.: Fatigue in Composites. Woodhead Publishing Ltd., Cambridge, England (2003).

4. Case, S.W., Reifsnider, K.L.: Fatigue of Composite Materials. In: Milne, I., Ritchie, R. O., Karihaloo, B. (eds.) Comprehensive Structural Integrity Volume 4: Cyclic Loading and Fatigue, pp. 405-442. Elsevier (2003).
5. Reifsnider, K.L.: Fatigue of Composite Materials. Composite Materials Series. Vol. 4. Elsevier Science, Amsterdam (1991).

6. Lemaitre, J., How to use damage mechanics. Nuclear Engineering and Design. 80, 233-245 (1984).

7. Lemaitre, J., and Dufailly, J.: Damage measurements. Engineering Fracture Mechanics. 28, 643-661 (1987).

8. Kachanov., L.M. Introduction to Continuum Damage Mechanics (Martinus Nijhoff Publishers, Dordrecht, The Netherlands (1986).

9. Jessen, S.M., Plumtree, A.: Continuum damage mechanics applied to cyclic behaviour of a glass fibre composite pultrusion. Composites. 22, 181-190 (1991).

10. Stigh, U.: Continuum damage mechanics and the life-fraction rule. Journal of Applied Mechanics. 73, 702-704 (2006).

11. Schapery, R.A.: On viscoelastic deformation and failure behavior of composite materials with distributed flaws. Advances in Aerospace Structures and Materials - AD-01 ASME, New York, 5-20 (1980).

12. Schapery, R.A.: Models for damage growth and fracture in nonlinear viscoelastic particulate composites. Proc. Ninth U.S. National Congress of Applied Mechanics, Book No. H00228, ASME, New York, 237-245 (1980).

13. Schapery, R.A.: Simplifications in the behavior of viscoelastic composites with growing damage. Proc. IUTAM Symp. on Inelastic DeJbrm. of Compos. Mater., Troy, NY, Springer, New York-Wien, 193-214 (1990).

14. Schapery, R.A.: Nonlinear viscoelastic constitutive equations for composites based on work potentials. Twelfth U.S. Nat. Congress of Appl. Mech., Appl. 47, 269-275 (1994).

15. Weitsman, Y.: A continuum damage model for viscoelastic materials, J. Appl. Mech. 55, 773-780 (1988).

16. Kim, Y.R., Little, D. N.: One-dimensional constitutive modeling of asphalt concrete. J. Eng. Mech. 116, No. 4, 751-772 (1990).

17. Park, S.W., Kim, Y.R., Schapery, R. A.: A viscoelastic continuum damage model and its application to uniaxial behavior of asphalt concrete. Mechanics of Materials. 24, 241-255 (1996).

18. Lee, H., Kim, Y. R.: Viscoelastic constitutive model for asphalt concrete under cyclic loading. Journal of Engineering Mechanics. 124, 32-40 (1998).

19. Schapery, R. A., Sicking, D. L.: On nonlinear constitutive equations for elastic and viscoelastic composites with growing damage. In: Bakker, A. (ed.) Mechanical Behaviour of Materials, pp. 45-76. Delft University Press, Delft, The Netherlands (1995).

20. Schapery, R.A.: Correspondence principles and a generalized J integral for large deformation and fracture analysis of viscoelastic media. Int. J. Fract. 25, 195-223 (1984).

21. Sullivan, R. W.: On the use of a spectrum-based model for linear viscoelastic materials. Mech Time-Depend Mater. 10, No. 3, 215-2008 (2006).

22. Eringen, C.A.: Mechanics of Continua. Wiley, New York (1967).

23. Kachanov, L. M.: On the time to failure under creep conditions. Izv. Akad. Nauk SSR, Otd. Tekhn. Nauk. 8, 26-31 (1958).