

Statistical Analysis of Modified Drucker-Prager Cap Model Parameters for Application to Modeling Die Compaction.

Gautam S. Wagle, Renata S. Engel, Ravi Bollina, and Randall M. German

**Department of Engineering Science and Mechanics
The Pennsylvania State University
University Park, PA 16802**

ABSTRACT

Many soil constitutive models have been studied for their applicability to the powder compaction simulation process. However most of the parameters to define these models are determined by high-pressure hydrostatic compression and triaxial tests requiring a high pressure setup. This limits the industrial usability of these simulations to determine density gradients in compacted powder parts for different powders. In order to overcome this limitation, we need to determine the extent particular parameters affect the prediction by the model. A statistical analysis is conducted here to determine the effect of the model parameters used in the modified Drucker-Prager "Cap" model and their interactions on the density predictions for a compacted part. A two-level factorial design has been developed to study the effect on the difference in the maximum and minimum predicted densities. The high and low levels for the model parameters have been selected to range most of the literature reported values for various metal powders. Based on this analysis, a regression model is proposed to describe the effect of model parameters on the predicted density gradients.

INTRODUCTION

Uniaxial die compaction has great potential for mass production of parts such as those in the automobile industry. However presence of die-wall friction leads to the presence of density gradients in the green part that translate to non-symmetrical dimensional changes during sintering (German, 1994). While die-wall friction can be reduced with the use of lubricants admixed with the powder or applied directly to the die-wall surface, it can never be eliminated. Hence it becomes essential to accurately predict density gradients developed in the green part.

Numerical simulation of the die compaction process using finite element analysis provides a means to achieve this goal. While one approach to numerical modeling of the process is to use micromechanical models (Cundall, 1979, Gethin, 1998) to model the particle-particle interaction, a

popular approach adopted by many researchers is the continuum modeling approach (Gurson, 1977, Mosbah and Bouvard, 1996, Weber and Brown, 1992, Trassoras *et al.*, 1992) with the proper selection of a material model to describe the complex nature of the process from loose aggregation of particles to a continuous skeleton structure. Not surprising, many soil constitutive models have been studied for their applicability as the material model in the numerical simulation of the die compaction process. Based on the results of the previous works related to studying the applicability of the Cam-clay model (Wagle *et al.*, 2000) and the modified Drucker-Prager “Cap” model (He *et al.*, 2001), the Cap constitutive model is used to define the material behavior during powder die compaction.

MODIFIED DRUCKER-PRAGER “CAP” MODEL

Originally intended to model cohesive geological materials that exhibit pressure-dependent yield, the Drucker-Prager Cap model represents the behavior of a material undergoing permanent deformation, thus making it a good candidate to model powder metal die compaction. Figure 1 shows the yield surface described by the Cap model in the p - q (hydrostatic pressure-deviatoric stress) plane (ABAQUS, 1998). The model is characterized by three principal segments: a pressure-dependent Drucker-Prager shear failure segment F_s , a compression cap F_c , and a transition surface F_t . Volumetric strain hardening is defined by moving the cap along the hydrostatic axis. The model has two purposes: to bound the yield surface in hydrostatic compression, thus providing an inelastic hardening mechanism to represent plastic compaction and to help control volume dilatancy when the material yields in shear by providing a softening law created as the material fails on the Drucker-Prager shear failure surface.

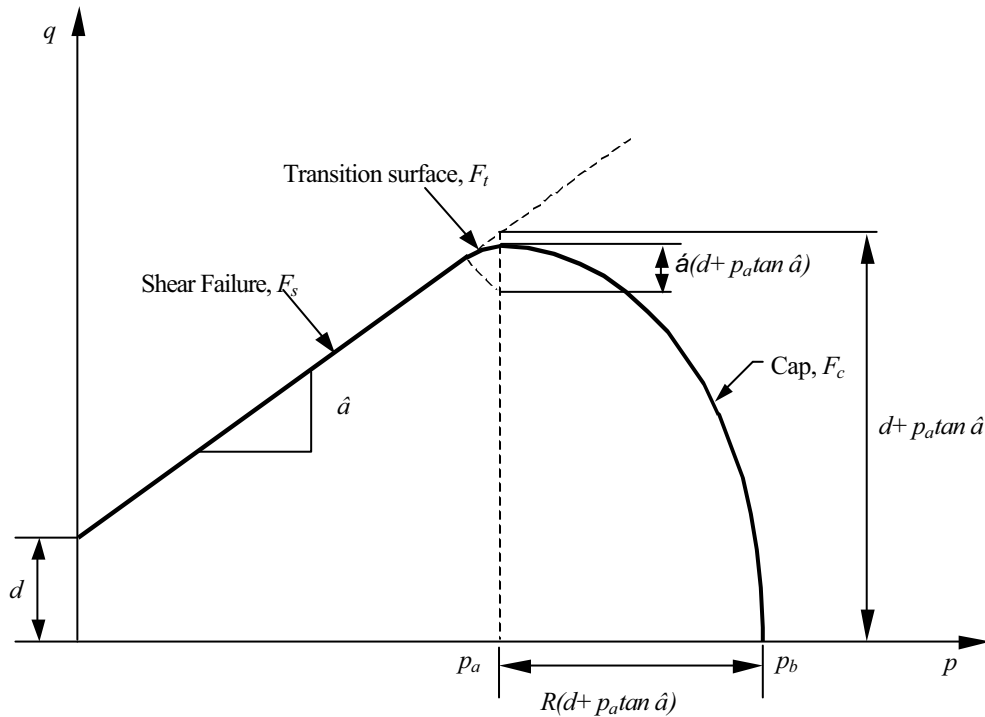


Figure 1. Yield surfaces defined by the modified Drucker-Prager Cap model

The failure surfaces in the meridional (p - t) plane are described by the shear failure surface,

$$F_s = t - p \tan \beta - d = 0 \quad \dots (1)$$

the cap failure surface,

$$F_c = \sqrt{[p - p_a]^2 + \left[\frac{Rt}{(1 + \alpha - \alpha/\cos \beta)} \right]^2} - R(d + p_a \tan \beta) = 0 \quad \dots (2)$$

and the transition failure surface,

$$F_t = \sqrt{[p - p_a]^2 + \left[t - \left(1 - \frac{\alpha}{\cos \beta} \right) (d + p_a \tan \beta) \right]^2} - \alpha(d + p_a \tan \beta) = 0 \quad \dots (3)$$

where β is the angle of friction, d is the material cohesion, t is the deviatoric stress measure, p is the equivalent pressure stress, R is the ratio of the horizontal axis of the elliptical cap to the vertical axis of the elliptical cap, $p_a(\epsilon_{vol}^{pl})$ represents the volumetric inelastic-strain-driven hardening and/or softening, and α represents the ratio of the radius of the transition surface to the radius of the vertical axis of the elliptical cap in the p - t plane.

The hardening/softening law is defined by a piecewise linear function relating the hydrostatic compression yield stress, p_b , to the volumetric inelastic strain as shown in Figure 2.

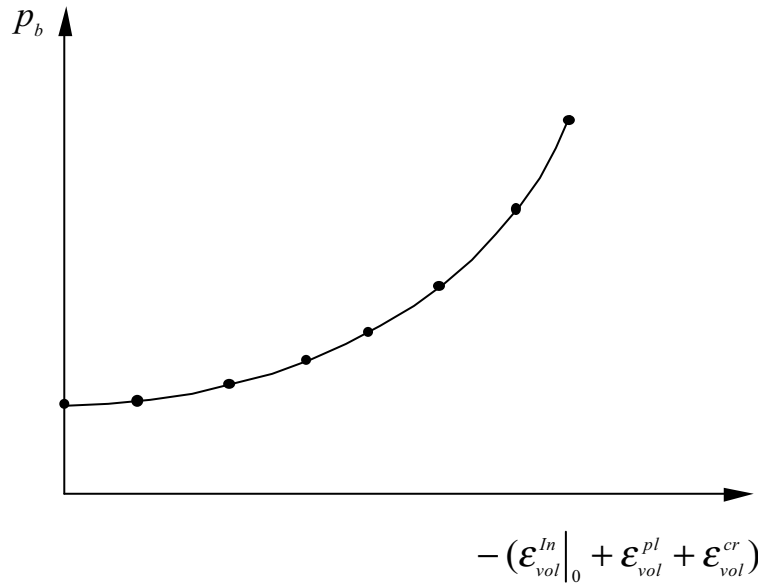


Figure 2. Cap hardening

The evolution parameter, p_a is related to p_b by

$$p_a = \frac{p_b - Rd}{(1 + R \tan \beta)} \quad \dots (4)$$

The definition of the modified Drucker-Prager cap plastic model thus includes plasticity parameters and hardening parameters. The plasticity parameters, $d, \beta, R, \epsilon_{vol}^{pl} |_0$ (initial volumetric plastic strain), α , and K define the shape of the yield surface in the stress space while the hardening law is defined by pairs of the hydrostatic compression yield stress and volumetric inelastic strain during the consolidation process.

DETERMINATION OF CONSTITUTIVE PARAMETERS

The cap plasticity parameters are usually obtained by triaxial tests (Meerman and Knaapen, 1979, Riera and Prado, 1994, Haggald *et al.*, 1996, Pavier and Doremus, 1996). The test begins with the specimen subjected to a hydrostatic state ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma$). To determine the parameters that define the shear failure surface, d and β , two of the principal stresses are kept constant ($\sigma_2 = \sigma_3 = \sigma$), while additional compression stress is applied in the remaining principal direction. Measurements are obtained in terms of the two stresses, σ_1 and $\sigma_2 = \sigma_3$, and deformation in the three orthogonal directions. As σ_1 increases, it will reach the critical state, where the volume will remain constant regardless of increases in σ_1 . The stress at the ultimate condition from each stress-strain curve at a different level of confinement is then plotted in the p - t plane.

To determine the cap eccentricity parameter, R that defines the cap yield surfaces, different stresses are applied in the three orthogonal directions. The three resulting principal stresses, σ_1 , σ_2 , σ_3 and the three principal logarithmic strains are measured with transducers to give the bulk densities and the corresponding stress states. At each value of density, the equivalent pressure stress, p , and deviatoric stress, t , are measured and iso-density curves plotted.

Though the triaxial testing is a straightforward and effective way to identify the cap plasticity parameters, triaxial experiment apparatus at high confining pressure for metal powder is expensive to build and not easily accessible. This limits the industrial ability to determine the constitutive model parameters for different grades and combinations of powders and hence the applicability of numerical simulations using these constitutive models to determine density gradients in powder compacts. Some alternative testing methods to achieve these parameters without having to achieve the confining pressures required in triaxial tests have been investigated. An alternative method using the combination of Brazilian disc tests and compression tests to measure the green strength of the powder compacts from the compression tests and the Brazilian disc tests has been used to determine the shear failure surface parameters, d and β (Coube and Riedel, 2000). However, the cap eccentricity parameter, R is known only to be determined using triaxial tests.

The initial cap yield surface position, $\epsilon_{vol}^{pl} |_0$ defines the initial cap position. In die compaction, the volume change is large due to the combined effects of particle rearrangement and plastic deformation. In order to prevent the green parts from falling apart after ejection, the relative density for the 316L stainless steel powder used in this research must be increased to 70%, which corresponds to an absolute value of the bulk volume strain of approximately 0.6. Thus usually a small number is chosen for the initial cap yield surface position, which is insignificant relative to the compacted parts. The transition surface radius parameter, α , is a small number (typically in the range of 0.01 to 0.05), and relates to the radius of the transition yield surface that provides a smooth intersection between the cap and failure surface. The ratio of the stress in triaxial tension to the flow stress in triaxial compression, K , is assumed to be 1 in this research.

The hardening law is a piecewise linear function relating the hydrostatic compression yield stress and the volumetric inelastic strain, $p_b - \epsilon_{vol}$. It describes the expansion of the yield surface for increasing volumetric strain. The hardening curve can be determined from the pressure-density relationship obtained

during uniaxial die pressing experiments. The hydrostatic pressure stress (absolute value), p_b , is related to axial stress, σ_z by the following relation (Zipse, 1997)

$$p_b = \frac{-(1 + R \tan \beta) \sigma_z}{1 + R \tan \beta \sqrt{1 + \left(\frac{2(1 + \alpha - \alpha / \cos \beta)}{3R} \right)^2}} \quad \dots (5)$$

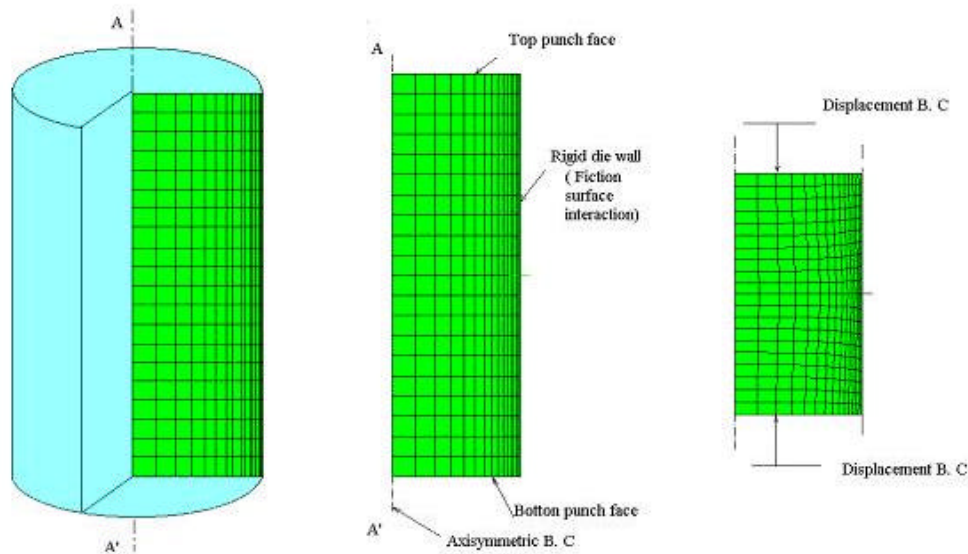
Since the elastic volumetric strain is much smaller compared to the plastic volumetric strain, the total volumetric strain, ϵ_{vol} , can be considered equal to the plastic volumetric strain, ϵ_{vol}^{pl} .

$$\epsilon_{vol} = \epsilon_{vol}^{pl} = \ln \left(\frac{\rho}{\rho_0} \right) \quad \dots (6)$$

where ρ is relative density in the current compacted parts, and ρ_0 is the initial (filling) relative density.

NUMERICAL SIMULATION AND SENSITIVITY ANALYSIS

A simple axisymmetric geometry was chosen for compaction. The specimen was modeled in the commercial software ABAQUS using a 300 element mesh made up of axisymmetric 8-noded bi-quadratic elements. The die wall was modeled as a rigid surface. A clearance of 0.001 mm was prescribed between the compact and die-wall. The friction between the boundary elements and the die wall was modeled as a friction surface interaction with a classical isotropic Coulomb friction model. Figure 3 shows the mesh before and after compaction along with the boundary conditions. Only half of the cylinder is modeled here to account for the symmetry about the longitudinal axis. Also note that when considering double acting pressing, the displacement boundary conditions are specified on both the top and bottom model surfaces. As described in the previous sections, the modified Drucker-Prager “Cap” model was used to describe the constitutive behavior of the powder material undergoing compaction.



(a) Cylindrical Model (b) Mesh before compaction (c) Mesh after compaction
Figure 3. ABAQUS FEM mesh to simulate the compaction process of an axisymmetric specimen

The parameters to describe the model in ABAQUS need to be experimentally determined using triaxial tests at high confining pressures which are difficult and expensive to set up. Hence the parameters that define the yield surfaces are studied via numerical simulation to determine their significance on some output function. This helps justify the need for these experiments. Sensitivity analysis of the parameters has been studied before but not in great detail.

He *et al.* (2001) studied the effect of the Cap plasticity parameters such as d , β , R and α on the density gradients achieved after compaction to show that the cap eccentricity parameter is the most significant in producing density gradients for a given set of parameters. However, the approach involved studying the effect of one-factor at a time. The effect of two factors interacting on the density gradient was not considered. Chtourou *et al.* (1996) used a series of Taguchi planned simulations to address friction-free uniaxial strain compression tests described by the Cap model. The relative influence was determined in terms of the contributing percentage and the Fisher factor, F of each parameter by applying the ANOVA (Analysis of Variance) method. They showed that the hardening parameters and the elastic moduli were most influential. While the design considered interaction effect of factors, the ANOVA method cannot be applied in numerical simulations since there is no error (or variance) between replications. In this study, a statistical analysis is conducted to determine the effect of the model parameters used in the modified Drucker-Prager "Cap" model and their interactions on the density predictions for a compacted part. A two-level factorial design has been developed to study the effect on the difference in field density achieved after compaction. A factorial design approach using a study of effects is considered to determine influence of parameters and based on this analysis a regression model is proposed to describe the effect of model parameters on the predicted density gradients.

FACTORIAL DESIGN APPROACH TO SENSITIVITY ANALYSIS

As mentioned, most of the sensitivity analysis in past considers a one factor at a time approach in which everything is held constant while only one factor is varied at a time. This is not representative of what happens in the real world. In a designed experimental approach (Montgomery, 2001), factors are allowed to vary simultaneously and the respective data is gathered and analyzed. This analysis can not only detect differences between the sample means, but the effect of the interactions between two or more variables. Many experiments involve the study of the effects of two or more factors. In such cases following types of factorial experiments are conducted to test effect of factors and their interactions.

- Full Factorial Designs - We can test for the effect of each factor at all levels of the other factors and can discover whether or not this effect changes as the other factors change. We can test not only for the effects of the factors separately, but also for interactions.
- Two-level Factorial Designs – As number of factors keep getting larger, running the experiments at all possible levels of all factors becomes expensive. In such cases, an initial factor screening experiment to determine which factors are important is a two-level factorial design. Here the k factors are each run at only two levels. Hence the size of the experiment is 2^k .
- Two-level Fractional Factorial Designs – If we can assume that the higher order interactions are negligible, information on the main effects and the low-order interactions can be obtained by running only a fraction of the complete factorial experiment to further cut down experimental costs. Major use of these experiments is to screen out the non-affecting factorials. Some of these experiments include half fractional (2^{k-1}), quarter fractional (2^{k-2}), etc.

Higher the fraction of the experiment, lower is the resolution of the design, i.e., its inability to predict if the effect is due to a main effect or an interaction and if an interaction, which one. It is therefore

crucial to select a fractional design such that the effect of the main effects and the lower order interactions such as the two factor interactions and the three factor interactions can be predicted clearly. In this research a study of the parameters that define the Cap model is conducted with their effect on predicting density gradients. As described in the previous sections, there are six plasticity parameters ($d, \beta R, \epsilon_{vol}^{pl} |_0$, α and K) and a hardening law described by $p_b v/s \dot{a}_{vol}$. The ratio of the stress in triaxial tension to the flow stress in triaxial compression, K , is assumed to be 1 in this research. Hence, our design has six factors. A two-level factorial design would require 64 runs in a 2^6 design to have all possible combinations of the factors. Since this is set as a screening experiment to find out which factors affect density gradient, we select a one-half fraction 2^{6-1} design of 32 runs. In this design all main effects and two-factor interactions are independent of other main effects and two-factor interactions. Three factor interactions are aliased with other three factor interactions. However they can be separated if necessary based on the Ockham's razor principle¹. The set-up of the numerical runs as obtained from the statistical software MiniTab is shown in table 1.

**Table 1. Half fractional (2^{6-1}) design with 32 numerical runs
(Note: -1 : Low value, 1: High value)**

Std. Order	Material Cohesion (d)	Angle of Friction (\hat{a})	Cap eccentricity parameter (R)	Hardening law ($p_b v/s \dot{a}_{vol}$)	Transition parameter (\hat{a})	Initial volumetric plastic strain ($\epsilon_{vol}^{pl} _0$)
1	-1	-1	-1	-1	-1	-1
2	1	-1	-1	-1	-1	1
3	-1	1	-1	-1	-1	1
4	1	1	-1	-1	-1	-1
5	-1	-1	1	-1	-1	1
6	1	-1	1	-1	-1	-1
7	-1	1	1	-1	-1	-1
8	1	1	1	-1	-1	1
9	-1	-1	-1	1	-1	1
10	1	-1	-1	1	-1	-1
11	-1	1	-1	1	-1	-1
12	1	1	-1	1	-1	1
13	-1	-1	1	1	-1	-1
14	1	-1	1	1	-1	1
15	-1	1	1	1	-1	1
16	1	1	1	1	-1	-1
17	-1	-1	-1	-1	1	1
18	1	-1	-1	-1	1	-1
19	-1	1	-1	-1	1	-1
20	1	1	-1	-1	1	1
21	-1	-1	1	-1	1	-1
22	1	-1	1	-1	1	1
23	-1	1	1	-1	1	1
24	1	1	1	-1	1	-1
25	-1	-1	-1	1	1	-1
26	1	-1	-1	1	1	1
27	-1	1	-1	1	1	1

¹ Ockham's Razor is the principle proposed by William of Ockham in the fourteenth century: "Pluralitas non est ponenda sine necessitate", which translates as "entities should not be multiplied unnecessarily". A more straightforward application of the Razor is when we are face with two theories which have the same predictions and the available data cannot distinguish between them. In this case the Razor directs us to study in depth the simplest of the theories.

Std. Order	Material Cohesion (d)	Angle of Friction (â)	Cap eccentricity parameter (R)	Hardening law (p_b v/s \dot{a}_{vol})	Transition parameter (â)	Initial volumetric plastic strain ($\epsilon_{vol}^{pl} _0$)
28	1	1	-1	1	1	-1
29	-1	-1	1	1	1	1
30	1	-1	1	1	1	-1
31	-1	1	1	1	1	-1
32	1	1	1	1	1	1

As a study, the actual values of the parameters were selected to cover the range of literature reported values for different grades of powders. Here the study is divided into two grades of powders depending on their hardening behavior: Metallic grade describing the hardening behavior for powders such as iron, steel, copper and Ceramic grade of powders describing the hardening law for powders such as alumina, tungsten carbide. The values are also based on certain limitations on parameters due to values of the other parameters. e.g. while the description of the transition parameter, \dot{a} , says it is a small number, it has a higher limit based on the material angle of friction imposed in ABAQUS due to geometric considerations. The output metric considered for the statistical analysis is defined as

$$\text{Output Metric} = \frac{(\text{Max Density} - \text{Min Density})}{\frac{1}{2}(\text{Max Density} + \text{Min Density})} \times 100 \quad \dots (7)$$

STATISTICAL ANALYSIS FOR METALLIC GRADE POWDERS

The hardening law is selected from pressure-density data corresponding to compaction for Hoeganaes A1000C iron powder and Höganäs 316L stainless steel water atomized powder each blended with 0.5% Acrawax C lubricant by weight for lubrication. The pressure density curves and the hardening law curves for the two powders are shown in figures 4 and 5 respectively.

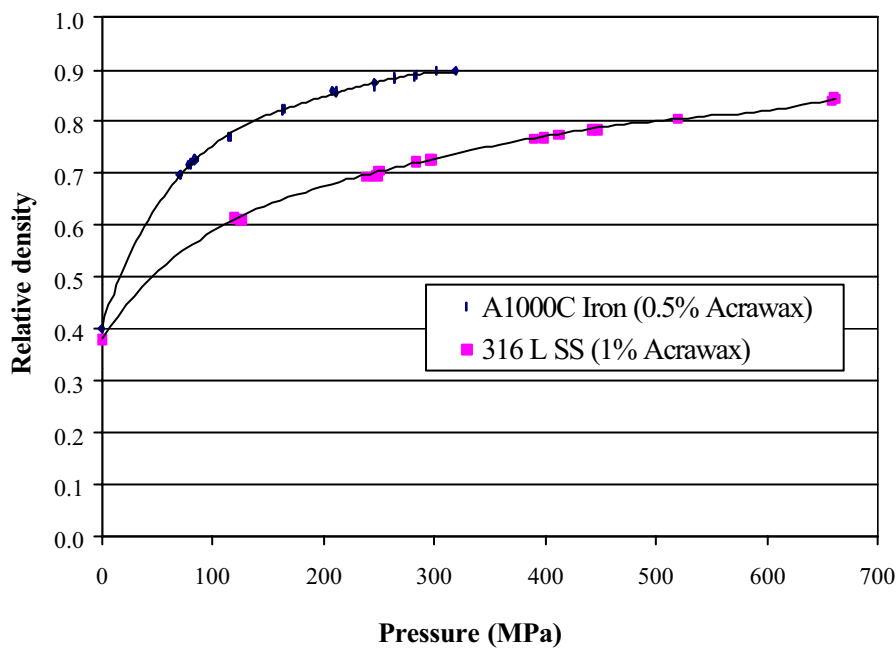


Figure 4. Pressure-density curves for A1000C iron powder and 316L stainless steel powder

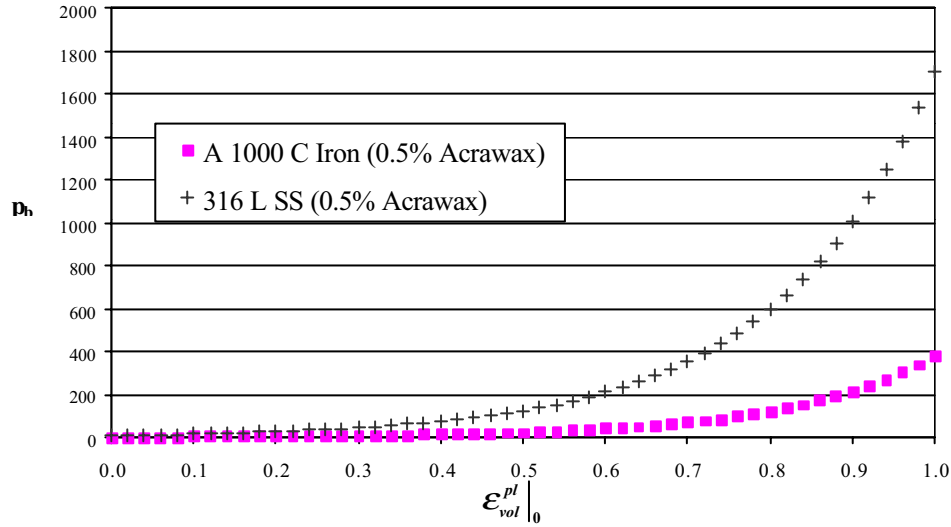


Figure 5. Hardening law curves for A1000C iron powder and 316L stainless steel powder

The entire listing of the low and high values of the parameters selected is shown in table 2.

Table 2. Low and high values of parameters for metallic grade of powders

Parameters	Term	Low value (-1)	High value (1)
• Material cohesion, d	A	0.01 MPa	1.0 MPa
• Material angle of friction, \hat{a}	B	60°	71.5°
• Cap eccentricity, R	C	0.1	0.87
• Hardening law, p_b v/s \hat{a}_{vol}	D	A 1000C (Fe)	316 L (St. steel)
• Transition surface radius parameter, \hat{a}	E	0.01	0.1
• Initial volumetric plastic strain, $\mathcal{E}_{vol}^{pl} \Big _0$	F	0.01	0.1

The normal effects plot of the output metric for the half-fraction factorial is shown in figure 6.

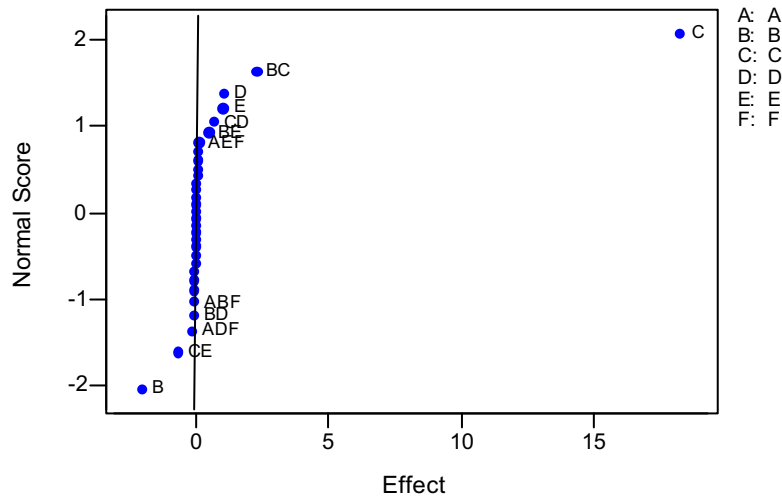


Figure 6. Normal probability plot of effects for metallic grade of powders

Based on the deviation from the normal effects line and the values of estimated effects, it is seen that the density gradient is sensitive to the following parameters in the following rank:

1. Cap eccentricity, R
2. Material friction angle, \hat{a}
3. Combination of R and \hat{a}

This implies that the cap eccentricity parameter and the material angle of friction are the most important parameters that affect the density gradient in a part after compaction and hence to accurately predict the density gradient using numerical simulations, it is essential that the values of R and \hat{a} be accurate for a given powder. This is consistent with the prediction of He *et al.* (2001) who indicate the importance of the eccentricity parameter on density difference. However due to a one factor at a time approach, the work is unable to predict the effect of the material angle of friction and its interaction effect with the eccentricity parameter on the density gradient.

STATISTICAL ANALYSIS FOR CERAMIC GRADE POWDERS

The hardening law is selected from pressure-density data corresponding to the compaction of alumina powder and tungsten carbide powder as described by the empirical relations given by Secondi (2002). The pressure density curves and the hardening law curves for the two powders are shown in figures 7 and 8 respectively.

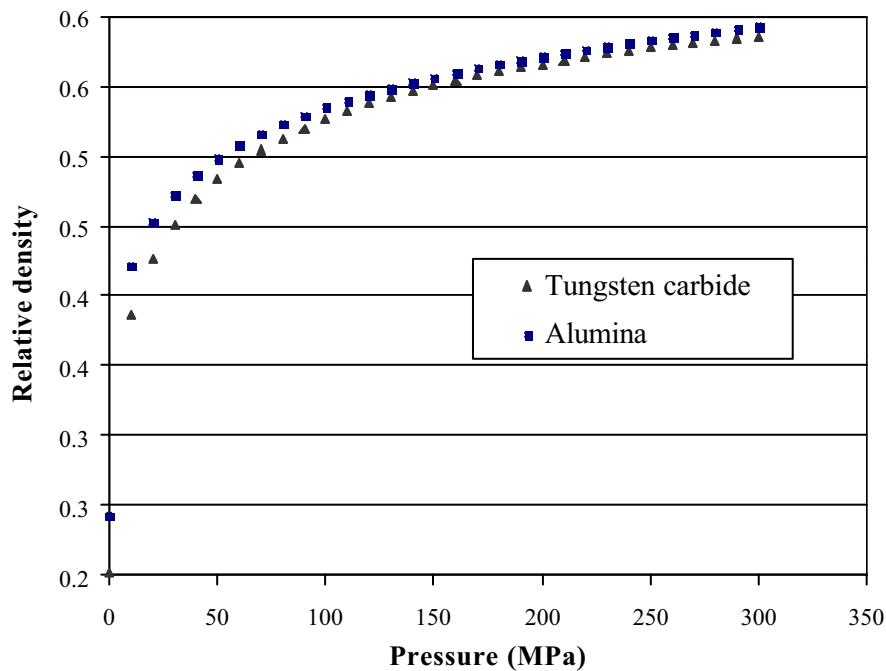


Figure 7. Pressure-density curves for alumina and tungsten carbide powder (Secondi, 2002)

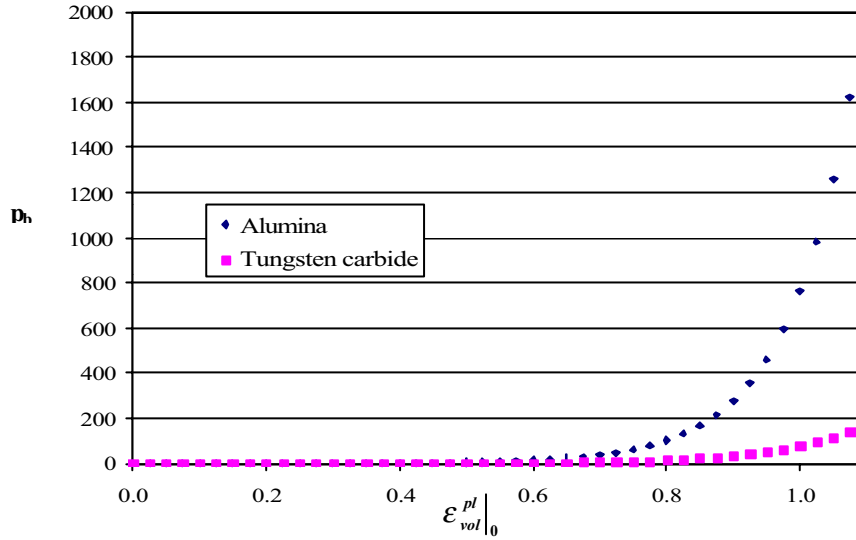


Figure 8. Hardening law curves for alumina and tungsten carbide powder

The entire listing of the low and high values of the parameters selected is shown in table 3.

Table 3. Low and high values of parameters for ceramic grade of powders

Parameters	Term	Low value (-1)	High value (1)
• Material cohesion, d	A	0.01 MPa	0.015 MPa
• Material angle of friction, $\hat{\alpha}$	B	60°	71.5°
• Cap eccentricity, R	C	0.1	0.87
• Hardening law, p_b v/s $\dot{\alpha}_{vol}$	D	Al ₂ O ₃	WC-Co
• Transition surface radius parameter, $\hat{\alpha}$	E	0.01	0.1
• Initial volumetric plastic strain, $\epsilon_{vol}^{pl} _0$	F	0.01	0.1

The normal effects plot of the output metric for the half-fraction factorial is shown in figure 9.

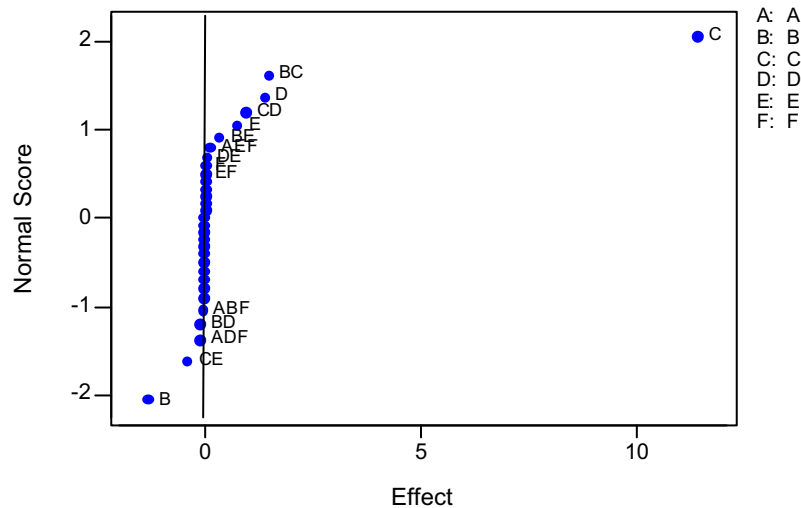


Figure 9. Normal probability plot of effects for ceramic grade of powders

Based on the deviation from the normal effects line and the values of estimated effects, it is seen that the density gradient is sensitive to the following parameters in the following rank:

1. Cap eccentricity, R
2. Material friction angle, \hat{a}
3. Combination of R and \hat{a}
4. Hardening data, p_b v/s \hat{a}_{vol}
5. Combination of R and p_b v/s \hat{a}_{vol}

Thus it is seen that the cap eccentricity parameter and the material angle of friction are the most important parameters that affect the density gradient similar to the predictions for the metallic grade of powders. In addition, the hardening law description also appears significant for the ceramic grade of powders. This might be because there is not much hardening for these powders and hence a difference in the hardening law affects the prediction.

CONCLUSIONS AND RECOMMENDATIONS

From the study it is seen that the sensitivity analysis using a designed two-level factorial approach is better than a one-factor at a time approach since it describes the effect of the individual parameters as well as the interaction between the parameters on the prediction of density gradient in a green part. The approach is necessary due to interlinking in the values of certain parameters based on those of other parameters.

The analysis for both the metallic and the ceramic grade of powders suggest that the plasticity parameters: the cap eccentricity parameter, R and the material angle of friction, \hat{a} are the most important parameters. Also significant for the ceramic grade of powders is the hardening law described in terms of p_b v/s \hat{a}_{vol} . These parameters can be achieved from triaxial tests as described earlier; however the cost of conducting these tests is what limits the industrial use of predicting tools. An alternative way could be predict a numerical model using regression analysis that predicts the output metric in terms of the parameters and then by experimentally measuring the output metric, the parameters can be back calculated for the particular powder. A simple linear regression model can be predicted based on the effects calculated during the sensitivity analysis. However a higher order model might be better suited that can be determined using a metamodel approach.

ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation under Grant No. 0200554.

REFERENCES

1. ABAQUS, "Users Manual", Version 5.8, 1998, Hibbit, Karlsson and Sorenson, Inc., Rhode Island.
2. Chtourou, H., Gakawaya, A., Guillot, M., "Assessment of the Predictive Capabilities of the Cap Material Model for Simulating Powder Compaction Problems", *Adv. In Powder Metallurgy & Particulate Materials*, Vol. 7, 1996, pp. 245 – 255.
3. Coube, O. and Riedel, H., "Numerical Simulation of Metal Powder Die Compaction with Special Consideration of Cracking", *Powder Metallurgy*, Vol. 43, 2000, pp. 123-131.

4. Cundall, P. A., Strack, O. D. L., "A Discrete Numerical Model for Granular Assemblies", *Geotechnique*, 1979, pp. 47 – 65.
5. German, R. M., *Powder Metallurgy Science*, 2nd Edition, Metal Powder Industries Federation, Princeton, NJ, 1994.
6. Gethin, D. T., Lewis, R. W., Ransing, R. S., "Compaction of Powder Via a Deformable Discrete Element Approach", *Proceedings of 1998 PM World Congress & Exhibition*, Granada, Spain, 1998, pp. 45 - 50.
7. Gurson, A. L., "Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I—Yield Criteria and Flow Rules for Porous Ductile Materials," *Jrnl. Of Engg Mat and Tech.*, Vol.99, 1977, pp. 2 - 15.
8. Haggbald, H. A., Doremus, P., Bouvard, D., "An International Research Program on the Mechanics of Metal Powder Forming", *Adv. In Powder Metallurgy &Particulate Materials*, Vol. 7, 1996, pp. 178 – 192.
9. He, Y., Engel, R.S., Salamon, N. J., Lindner, S., Lu, P., "Numerical Simulation for 316L Stainless Steel Powder Die Compaction Process", *Proceedings of the International Conference on Fine Powder Processing Conference*, Ed., J. H. Adair, V. M. Puri, K. S. Haris, C. C. Huang, 2001, pp. 153-166.
10. Meerman, W. C. P. M., Knaapen, A. C., "A High-Pressure Triaxial Testing Cell", *Powder Technology*, Vol. 22, 1979, pp. 271 – 278.
11. Montgomery, D. C., *Design and Analysis of Experiments*, 5th Ed., John Wiley, NY, 2001.
12. Mosbah, P., Bouvard, D., "Finite Element Simulation of Die Compaction and Ejection of Iron Powder", *Adv. in Powder Metallurgy & Particulate Materials*, Vol. 7, 1996, pp. 23 – 37.
13. Pavier, E., Doremus, P., "Mechanical Behavior of a Lubricated Iron Powder", *PM96*, 1996, pp.27–40.
14. Riera, M. D., Prado, J. M., "Modeling of Cold Compaction Process of Metal Powders," *PM94*, 1994, pp. 685-688.
15. Secondi, J., "Modeling Powder Compaction from a Pressure-Density Law to Continuum Mechanics", *Powder Metallurgy*, Vol. 45, no. 3, 2002, pp. 213 – 217.
16. Trassoras, J., Krauss, T. M., Fergusson, B. L., "Modeling of Powder Compaction Using the Finite Element Method", *Adv. in Powder Metallurgy*, Vol. 1, 1992, pp. 85 – 104.
17. Wagle, G. S., Engel, R. S., German, R. M., Liu, Y., "Numerical Investigation of Constitutive Models for the Powder Compaction Process", *Adv. In Powder Metallurgy &Particulate Materials*, Vol. 1, 2000, pp. 13 – 24.
18. Weber, G. G., Brown, S. B., "Simulation of Compaction of Powder Components", *Adv. in Powder Metallurgy*, Vol. 1, 1992, pp. 105 – 188.
19. Zipse, H., "Finite-element Simulation of the Die Pressing and Sintering of a Ceramic Component" *Journal of the European Ceramic Society*, Vol. 17, 1997, pp. 1707-1713.