Textbook title: Circuits, Devices, Networks and Microelectronics

CHAPTER 19. NOISE ANALYSIS AND LOW-NOISE DESIGN

19.1 THE ORIGINS OF NOISE

Electrical noise is a background "fuzz" of unwanted signals, usually due to thermal origins. It has a nearly constant amplitude density across the frequency spectrum that tends to mask and obscure the waveforms and information which we wish for our circuits to process.

Noise is an inescapable fact of circuits and signals. It is generated, in most part, by thermal fluctuations. It is an important factor in the design and analysis of communication circuits, and therefore most treatments of noise are developed in this context. But electrical noise, and its companion problem, distortion, are an important and necessary consideration of any circuit, in which a signal, whether of linear or logic form, is to be processed.

We are interested in defining the range of input signals over which a circuit is "good". A figure of merit is the dynamic range (DR) given by

$$DR = \left[\frac{\text{largest usable signal}}{\text{smallest usable signal}}\right]$$

The smallest usable signal level is defined by the *noise limit*. The largest usable signal is defined by the *distortion limit*, which is usually a consequence of the compliance limits of the circuit.

Although we seldom regard the molecular or atomic levels in dealing with circuit electronics, we realize that they are driven by thermal processes. And thermal statistical fluctuations will produce a random background of signals in an electrical component. These thermal effects manifest themselves in fluctuations in electrical currents. The basic unit of thermal energy (fluctuation) is given by $\Delta w = kT$ (also called the fugacity), where k = Boltzmann's constant and T = absolute temperature. Translate that into the fluctuations in power and we have

$$\Delta p = \Delta w \times B = kT \times B \qquad (19.1\text{-}2a)$$

where B = bandwidth of the frequencies passed by the circuit. Since power usually relates to signal through resistance then the (rms) noise thermal power is

$$\Delta p = e_n^2 / R = i_n^2 G \qquad (19.1\text{-}2b)$$

where R = resistance and G = conductance. These give the noise signals in terms of e_n = noise voltage and i_n = noise current. The maximum signal that can be passed from the source, which is presumed to be the resistance itself, (according to maximum power transfer) is then

$$e_n^2 = 4kTRB \qquad (19.1\text{-}3a)$$

or

$$i_n^2 = 4kTGB \qquad (19.1\text{-}3a)$$

Equation (19.1-3) is called *Nyquist's theorem*. Note that the measure is in terms of a *root mean square* noise voltage (or current).

Consider the following example:

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EXAMPLE 19.1-1: What is the amplitude e_n of the thermal noise associated with a resistance R = 1kΩ at a noise bandwidth B = 1 MHz?

SOLUTION: $$e_n^2 = 4kTRB = 4 \times \frac{kT}{q} \times q \times R \times B$$

$$= 4 \times .025V \times (1.602 \times 10^{-19} C) \times 10^3 \Omega \times 10^6 s^{-1}$$

$$= (1.602 \times 10^{-20}) \times 10^9 V^2 \qquad = 16 \times 10^{-12} V^2$$

Note that we have taken advantage of the fact that $kT/q = V_T$ (= thermal voltage) $\approx .025V$ at (nominal) circuit temperature 300K.

And take note that $4kT \cong 1.6 \times 10^{-20} J$. This may be a handy simplification for other situations involving noise analysis.

Taking the square root we get noise amplitude $e_n = 4\mu V$

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Equations (19.1-3a) and (19.1-3b) may be represented by the network equivalents of figure 19.1-1

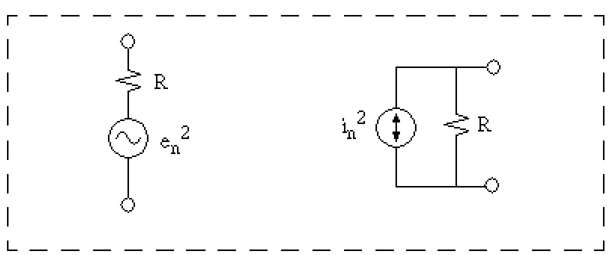

Figure 19.1-1: Equivalent circuits for thermal noise

This type of noise is also called *Johnson* noise, or *white* (full spectrum) noise since it is spread uniformly across all frequencies of the spectrum. And it is called thermal noise since it is a result of thermal statistics.

Even though we could think of each resistance contributing noise via its own little noise source and can analyze the network as if it were made up of a mess of noise sources and resistances, we would quickly find that this is a challenging but totally useless exercise. Thevenin theorem works just fine in our behalf. The only reason that we might ask the question would be for sensitivity analysis, and that is more of a task for circuit simulation software.

However, the assessment of a network, which may contain both resistances and reactive components such as capacitances and inductances does imply the need for an expanded definition of equation (19.1-3). This expanded form is called the *Nyquist formula* and is given by

$$e_n^2 = 2kT\int_{-\infty}^{+\infty} R(f)df \qquad = 4kT\int_{0}^{+\infty} R(f)df \qquad (19.1\text{-}4)$$

where *R(f)* is the real part of the impedance, *Z(f)*, as seen across the output nodes.

Nyquist's formula can also be written as

$$e_n^2 = 4kTR\int_{0}^{+\infty} G(f)df \qquad = 4kTRB$$

for which *G(f)* = (frequency dependent) *power gain* = v_{out}^2/v_{in}^2, assuming the resistance term *R* is separable. And it gives a clear definition of the noise bandwidth.

Consider the following example:

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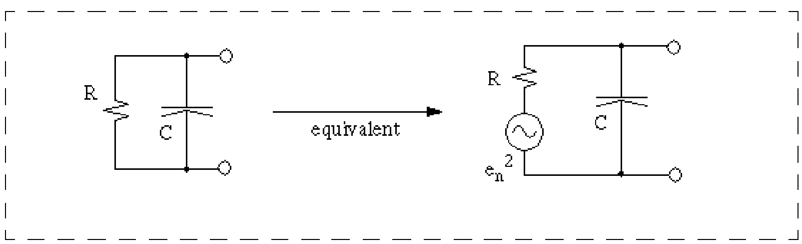

Figure 19.1-2: Single-time constant *RC* network

EXAMPLE 19.1-2 Consider the single-time constant *RC* network shown by figure 19.1-2. Using Nyquist formula find (a) noise signal e_n^2 and and (b) noise bandwidth *B*.

The impedance, looking into the output is $Z = \dfrac{1}{G + sC} = \dfrac{R}{1 + j\omega RC}$

and its real component is $\quad \mathrm{Re}[Z(f)] = \dfrac{R}{1 + \omega^2 R^2 C^2} = R(f)$

Therefore, using Nyquist's formula, equation (19.1-4) gives

$$e_n^2 = 4kT\int_0^{\infty} \frac{R}{1+\omega^2R^2C^2}df = \frac{4kT}{2\pi C}\int_0^{\infty}\frac{RCd\omega}{1+\omega^2R^2C^2}$$

$$= \frac{4kTR}{2\pi}\times\frac{1}{RC}\times\frac{\pi}{2} \qquad = 4kTR\times\frac{1}{2\pi RC}\times\frac{\pi}{2}$$

and therefore the noise bandwidth $B = \frac{\pi}{2} \times f_{3dB}$

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Note that the noise bandwidth is greater than the (STC) 3dB bandwidth by the factor π/2. We generalize this characteristic to all circuit networks as

$$\textit{Noise bandwidth} = B = \frac{\pi}{2} \times f_{3dB} = \frac{1}{4}\omega_{3dB} \qquad (19.1\text{-}5)$$

19.2 SYSTEM NOISE ANALYSIS

Noise is a form of background signal. In this respect it should be expected that the detector/transducer at the front end of the circuit is the major source of the noise, in terms of some sort of 'dark current' signal, with characteristics as might be represented by the source resistance R_S. But it is also true that the components within the amplifier/interface circuit also contribute noise, and it is the assessment of the system noise factors that is necessary to identifying the noise characteristics of the system.

The comparison between the desired signal and the noise signal are defined quantitatively by the signal-noise ratio

$$\frac{S}{N} = \frac{(\text{signal power})}{(\text{noise power})} \qquad (19.2\text{-}1)$$

Both signal S and noise N are amplified by the gain factor G of the amplifier. But the noise N_O at the output is greater than the amplified input noise = GN_i, courtesy of additional noise contributed by the components within the amplifier, i.e.

$$N_O = GN_i + N_{Oa} \qquad = GN_i + GN_a = G(N_i + N_a) \qquad (19.2\text{-}2)$$

where N_{Oa} = additional noise at the output = GN_a. It is as if an additional noise source Na exists at the input. We usually call ($N_i + N_a$) the equivalent noise input, and it gives us a means to define the principal noise characteristic of the system, the noise factor = F, which we define as the ratio

$$F = \frac{N_i + N_a}{N_i} \qquad = 1 + \frac{N_a}{N_i} \qquad (19.2\text{-}3)$$

for which we might take note that F is always greater than unity.

We can modify equation (19.2-3) to included the power gain $G = S_O/S_i$, where S_O and S_i represent the signal levels at output and input, respectively of the system, i.e.

$$F = \frac{N_O}{GN_i} \qquad = \frac{N_O}{(S_O/S_i)N_i} \qquad = \frac{S_i/N_i}{S_O/N_O} \qquad (19.2\text{-}4)$$

so the noise factor also represents the degradation of the *S/N* due to the system.

We should take note of the fact that the signal-to-noise and noise factor are in terms of signal power. The maximum signal power transferred and consequently the maximum available noise power per bandwidth occurs when $R_{in} = R_S$, for which

$$N_i = \frac{e_n^2}{4R_S} = \frac{4kTR_S B}{4R_S} = kTB \qquad (19.2\text{-}5)$$

for which

$$F = 1 + \frac{N_a B}{kTB} \qquad = 1 + \frac{T_e}{T} \qquad (19.2\text{-}6)$$

if we assume that the system has some sort of equivalent noise temperature T_e, in which case we can use this as a benchmark as an alternative to the noise factor *F*. Take note that

$$T_e = (F - 1) \times T \qquad (19.2\text{-}7)$$

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Example 19.2-1: Noise factor F = 2.5 corresponds to

$$T_e = (2.5 - 1) \times 300 \;= 450\text{K}$$

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(Note that we assume nominal circuit temperature *T* to be approximately 300K).

Since we are ultimately interested in defining the dynamic range of the system, we have need to identify a lower limit, or smallest usable, signal. We define this limit as the *noise floor* $N_f = S_i(min)$ needed to achieve a given output *S/N*. A corollary to this definition is the *minimum detectible signal*, which is the input signal voltage $v_i(min)$ associated with the noise floor, for which, assuming that $S_i = v_i^2 / R_S$, then the minimum detectible signal is

$$v_i(\min) = \sqrt{4R_S S_i(\min)} \qquad (19.2\text{-}8)$$

From the definition (19.2-4) of noise factor in terms of S/N ratios, and assuming the systems are matched, (for which $R_{in} = R_S$) then

$$N_f = S_i(\min) = N_i \times F \times \frac{S_O}{N_O} = kTB \times F \times \frac{S_O}{N_O} \qquad (19.2\text{-}9)$$

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EXAMPLE 19.2-2: What is the noise floor and minimum detectible signal for a system with bandwidth 20kHz, noise factor 8dB, input resistance R_{in} = 50Ω if the output signal/noise requirement is 10dB.

SOLUTION: Using equation (19.2-9) we get

$$10\log S_i(\min) = 10\log(kTB) + 10\log F + 10\log(S_O/N_O)$$

$$= 10\log(kT) + 10\log B + 10\log F + 10\log(S_O/N_O)$$

= -143dBW

Take note: In the assessment of such levels of power we often use dB measure taken relative to 1.0W. And hence the units of measure are in 'dBW'. And that is why we often will exercise the analysis in terms of dB measure and define the noise factor in terms of dB. In some references this usage is called the '*Noise Figure* and is abbreviated as 'NF'.

If analysis is done using logarithmic arithmetic, it is usually convenient to make use of the shortcut

$$10\log(kT) = 10\log(0.4\times 10^{-20}) = -204dBW$$

The value for the noise floor is then either in terms of dBW, or of the same form with respect to the 1 milliwatt reference level. For this option we would have noise floor value

$$S_i(\min) = N_f = -143dbW = -113dBm$$

This may be a more dialogue than what we really need, but it is necessary to become acquainted with the units of measure associated with very low RF power levels, as should be expected when speaking the language of noise.

The power level of the noise floor (*Nf*) in watts (via the antilog operation is then

$$S_i(\min) = 5\times 10^{-15}W = 5.0\,fW$$

for which, using equation (19.2-9) we get

$$v_i(\min) = \sqrt{4R_S S_i(\min)} = \sqrt{4\times 50\times(5\times 10^{-15})} = 1\times 10^{-6}V$$ = 1.0uV

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Note: There is nothing wrong with using non-logarithmic arithmetic. In this case it was applied in order to highlight and identify the logarithmic units of measure, dBW and dBm.

19.3 CASCADED STAGES

A signal conditioning circuit, particularly those associated with RF applications, generally consists of several stages in series, some of which do nothing more than serve as buffers, linking cables, or filters, as well as those which we associate with signal gain, as may be needed to convert the signal to a useful form. Each of these stages contributes noise and has a noise factor and a gain (or loss). The aggregate effect of the system is an equivalent input noise N_a of the same form as identified by equation (19.2-3).

Assuming that a system can be identified in terms of a set of stages, as represented by figure 19.3-1, we can evaluate the aggregate noise character in terms of the separate noise and gain characteristics by a systematic approach.

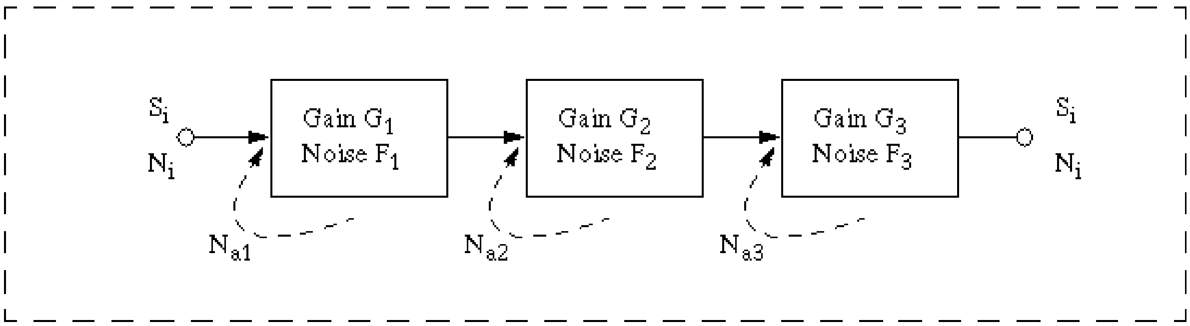

Figure 19.3-1: Cascaded stages

For the first stage the equivalent input noise due to its characteristics specified by equation (19.2-3) is

$$N_{a1} = N_i(F_1 - 1) \qquad (19.3\text{-}1)$$

and similarly the equivalent input noise to the second stage due to its noise character is , etc $N_{a2} = N_i(F_2 - 1)$. And when this noise is related to the input of stage one it can be assumed to be equivalent to an input noise at this point of value

$$N'_{a2} = \frac{N_{a2}}{G_1}$$

and similarly, the equivalent input noise due to Na3 as related to an equivalent source at the input would be

$$N'_{a3} = \frac{N_{a3}}{G_1 G_2}$$

and therefore the entire equivalent input noise due to the three stages would be

$$N_a = N_{a1} + N'_{a2} + N'_{a3} = N_i(F_1 - 1) + \frac{N_i(F_2 - 1)}{G_1} + \frac{N_i(F_3 - 1)}{G_1 G_2} \qquad (19.3\text{-}2)$$

Since $F = (N_i + N_a)/N_i$ then the overall system noise figure is

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} \qquad (19.3\text{-}3)$$

The context can be extended to any number of stages in like manner

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \frac{(F_4 - 1)}{G_1 G_2 G_3} + \cdots \qquad (19.3\text{-}4)$$

and in like manner, since $T_{e1} = (F_1 - 1)T$, then we can also restate equation (19.3-4) in terms of noise temperature, for which

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e2}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \cdots \qquad (19.3\text{-}5)$$

and which is a little simpler.

Consider the following example:

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EXAMPLE 19.3-1: A receiving antenna that has noise temperature 450K is connected to an amplifier that has noise figure 3dB and gain 10dB by a UHF cable that suffers a loss of 0.2dB per m and has noise figure 2dB. The cable is 5m long.

(a) What is the overall system noise factor and (noise) temperature of the system?
(b) What is the noise factor and temperature of the system if a noiseless preamplifier of gain 6dB is placed right after the antenna (preceding the connecting cable.

ANALYSIS:

Antennas have no gain or loss. They just pick up signal. So the gain of the antenna $G_1 = 1$. If the antenna is not particularly selective it will pick up extraneous signal, which is manifested as a noise temperature. In this case T_1 = 450K. And this T_e is the same as $F_1 = 1 + 450/300 = 2.5$.

The cable has thermal properties that manifest themselves as a noise. A noise figure of 2dB corresponds to noise factor $F_2 = 1.585$ [note: $F = \log^{-1}(2/10)$]. And this corresponds to noise temperature $T_2 = (1.585 - 1) \times 300 = 175$K.

The cable has a loss of 5m x (-0.2dB/m) = -1dB, which corresponds to factor $G_2 = 0.79$.

Notice that we *always* should determine the noise factor F, noise temperature T_e, and gain (loss) of each part of a system.

The amplifier has noise factor $F_3 = 2.0$ (same as noise figure 3dB) (same as noise temperature 300K) and gain $G_3 = 10$.

(a) Now put all these together in the order indicated, for which

$T_1 = 450$, $F_1 = 2.5$, $G_1 = 1.0$
$T_2 = 175$, $F_2 = 1.585$, $G_2 = 0.79$
$T_3 = 300$, $F_3 = 2.0$, $G_3 = 10$

And therefore $F = 2.5 + \dfrac{(1.585 - 1)}{1.0} + \dfrac{(2.0 - 1)}{1.0 \times 0.79}$ = 4.35

= 1005K

And this is the same as $T_e = (4.35 - 1) \times 300$

It is usually simpler to find T_e (rather than *F)*, i.e.

$$T_e = 450 + \frac{175}{1} + \frac{300}{1.0 \times 0.79} \qquad \boxed{= 1005\text{K}}$$

(a) If we insert a preamplifier for which $G = 4.0$ (same as 6dB), $F = 1$ ($T_e = 0$) after the antenna, then the order becomes one of four stages

$T_1 = 450$, $F_1 = 2.5$, $G_1 = 1.0$
$T_2 = 0$, $F_2 = 1{,}0$ $G_2 = 4.0$
$T_3 = 175$, $F_3 = 1.585$, $G_3 = 0.79$
$T_4 = 300$, $F_4 = 2.0$, $G_4 = 10$

and $$T_e = 450 + \frac{0}{1} + \frac{175}{1 \times 4} + \frac{300}{1.0 \times 4 \times 0.79} \qquad \boxed{= 588\text{K}}$$

which is a considerable improvement.

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Take note that a common means of improving the noise factor (or temperature) of a system is to include a low-noise preamplifier at the front-end.

19.4 ANALYSIS AND DESIGN FOR LOW-NOISE ELECTRONICS

When we assess the noise contributed by a system we always treat it as if it were an equivalent input signal, with the system itself as being little more than a gain factor. That gives us the simple context of noise factor and noise temperature.

System noise needs to be realized both in terms of current fluctuations and voltage fluctuations, as represented by figure 19.4-1. e_n^2 and i_n^2 are both needed, since system noise will exist even with a short circuit or an open circuit of the source.

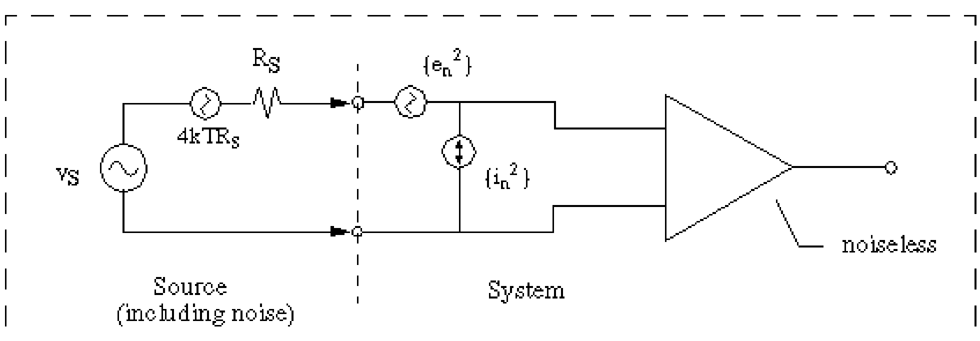

Figure 19.4-1: Generalized representation of amplifier noise

A nomenclature is also represented by figure 19.4-1, namely that of the noise contribution as a *noise-spectral density*, i.e. noise signal per bandwidth in either V^2/Hz or A^2/Hz. This context is not all that necessary with resistances, since such noise is adequately and succinctly represented by equation (19.1-3), but it is necessary with transistors, since they are dominated by *shot noise*, which is a noise/Hz.   Shot noise has an noise spectral density (NSD) of the form

$$\{i_n^2\} = 2qI \qquad (19.4\text{-}1)$$

and usually is <u>not</u> uniform over the frequency spectrum. Usually it is greater at low frequencies. And because of that fact it is usually called *pink noise* in comparison to that of thermal noise (which we also call white noise). The noise contributions vary as *1/f* and are modeled in circuit simulators by

$$\{i_{nf}^2\} = K_f \frac{I^a}{f^b}\Delta f \qquad (19.4\text{-}2)$$

This type of noise is supported by our old friend SPICE. Usually b = 1. SPICE uses AF for a and KF as indicated.

If we do flip over to the usage of NSD for resistance, it is just a matter of dividing equation (19.1-3) by noise bandwidth B, for which

$$\{e_n^2\} = 4kTR \qquad (19.4\text{-}3a)$$

$$\{i_n^2\} = 4kTG\,. \qquad (19.4\text{-}3b)$$

Circuit simulators use NSD since the frequency is considered an axis in noise analysis. When checked out via SPICE, to no great surprise you will find that $\{e_n^2\}$ = 1.657 x 10^{-20} for R = 1Ω, which is what you should expect for T = 300K and k = 1.3806 x 10^{-23}J/K. (Do the math).

Note: The spice nomenclature for $\{e_n^2\}$is *NTOT(RS).* And a SPICE exercises to this effect accompany these notes.

And via the NSD nomenclature we can break out the system noise for a circuit and tailor its design for minimum noise contribution to the system.

Notice that the noise factor does not care about the bandwidth, since it is a common factor to both numerator and denominator as identified by equation (19.2-3), for which

$$F = \frac{N_i + N_a}{N_i}$$

Now if we make use of the model of figure 19.4-4 to do a break out of N_a into system-level terms, then

$$N_a = \{e_n^2\} + \{i_n^2\}R_S^2 \qquad (19.4\text{-}4)$$

And the noise factor becomes

$$F = \frac{4kTR_S + \{e_n^2\} + \{i_n^2\}R_S^2}{4kTR_S} \qquad = 1 + \frac{\{e_n^2\} + \{i_n^2\}R_S^2}{4kTR_S} \qquad (19.4\text{-}5)$$

Now we have a platform that we can use to assess the various contributions to *F*. For example, minimizing *F* with respect to R_S, we have

$$\frac{\partial F}{\partial R_S} = \frac{1}{4kT}\left[-\frac{\{e_n^2\}}{R_S^2} + \{i_n^2\} \right] = 0$$

for which, using [ ] = 0, we have $$R_S\,(optimum) = \sqrt{\frac{e_n^2}{i_n^2}} \qquad (19.4\text{-}6)$$

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EXAMPLE 19.4-1: Opamps are packaged IC components that are characterized as systems, much like that indicated by figure 19.4-1. Their noise characteristics are frequency dependent and lend themselves to a graphical representation of $\{e_n^2\}$ and $\{i_n^2\}$, or more likely the square roots of these terms, since they are very aware of equation (19.4-6). These characteristics are represented by figure E19.4-1, for the HA2406 general-purpose opamp.

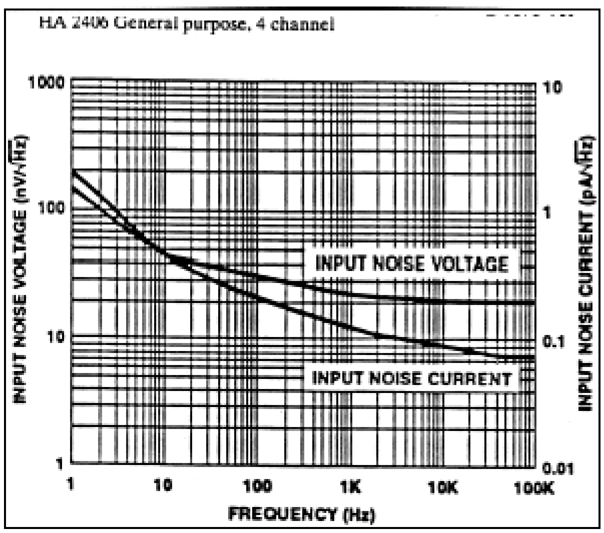

Figure E19.4-1: Noise characteristics of the HA2406 general purpose opamp

The left-hand axis identifies the NSD for equivalent noise voltage $\{e_n\} = \sqrt{\{e_n^2\}}$ and (it should be noted that the units are $V/\sqrt{Hz}$). The right-hand axis identifies the NSD for equivalent noise current $\{i_n\} = \sqrt{\{i_n^2\}}$ (and it should be noted that the units are $A/\sqrt{Hz}$). The vertical axes are logarithmic, which should not be unexpected. For higher frequencies than those shown the $\{e_n\}$ and $\{i_n\}$ are approximately the same as the right-most values.

Exercise requirement: (a) Find the optimum source resistance for an application that uses the HA2406 at carrier frequency 20kHz (carry-current applications operate at these frequencies) and the noise factor F and noise temperature T_e that results. (b) find F and T_e if a source with R_S = 50kΩ is used.

(a) From figure E19.4at the 20kHz point on the x-axis we find: $\{e_n\} \approx 20\, nV/\sqrt{Hz}$ and $\{i_n\} \approx .085\, pA/\sqrt{Hz}$. (Note that you have to extract these from logarithmic measures, which means that your accuracy will not be all that good.)

So $R_S(opt) \cong \dfrac{20\, nV/\sqrt{Hz}}{.085\, pA/\sqrt{Hz}} \qquad \cong 235k\Omega$

For which $4kTR_S$ = $1.657 \times 10^{-20} \times 235 \times 10^3 = 38.9 \times 10^{-16}$ and the noise factor is

$$F = 1 + \frac{20^2 \times 10^{-18} + (.085 \times 235)^2 \times 10^{-18}}{38.9 \times 10^{-16}} = 1 + \frac{4+4}{38.9} = \boxed{1.21}$$

and this correspondes to a noise temperature of T_e = (1.21 -1) x 300 = $\boxed{63K}$

(b) For R_S = 50kΩ, then $4kTR_S$ $= 1.657 \times 10^{-20} \times 50 \times 10^3 = 8.285 \times 10^{-16}$

and using the same values for $\{e_n\}$and $\{i_n\}$ as before, the noise factor is

$$F = 1 + \frac{20^2 \times 10^{-18} + (.085 \times 8.285)^2 \times 10^{-18}}{8.285 \times 10^{-16}} = 1 + \frac{4 + 0.704}{8.285} = \boxed{1.57}$$

and this corresponds to a noise temperature of T_e = (1.57 -1) x 300 = $\boxed{170K}$

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The transistors themselves tell the story about the noise characteristics $\{e_n^2\}$ and $\{i_n^2\}$. They include bias resistances, which will have noise contributions. There are two ways in which we can diagnose the effects of noise contributions within a circuit (a) by reflecting the noise back to an equivalent input or (b) by accumulating all of the noise effects at the output in the form of $\{e_n^2\}$ and then relate that to its input equivalent by means of $\{e_n^2\}/A_V^2$.

The transistor noise terms are of the form of (a) shot noise current noise and (b) noise due to parasitic resistances.

For the BJT shot noise relate to base currents and collector currents according to equation (19.4.1), as

$$\{i_{nB}^2\} = 2qI_B \qquad (19.4\text{-}7a)$$
$$\{i_{nC}^2\} = 2qI_C \qquad (19.4\text{-}7b)$$

In addition there is a parasitic resistance, called the base-spreading resistance r_B, for which

$$\{e_{nB}^2\} = 4kTr_B \qquad (19.4\text{-}8)$$

For the FET, shot noise relates to the square root of the drain current, and consequently (and most usually) is expressed in terms of the transconductance, g_m

$$\{i_{nD}^2\} = \frac{2}{3} \times (4kT) \times g_m \qquad (19.4\text{-}9)$$

Circuit simulation can accomplish these embedded noise contributions very handily, but for benefit of the process and a generic and simple relationship which can be seen with one of the generic transistor topologies, consider the circuit shown by figure 19.4-3.

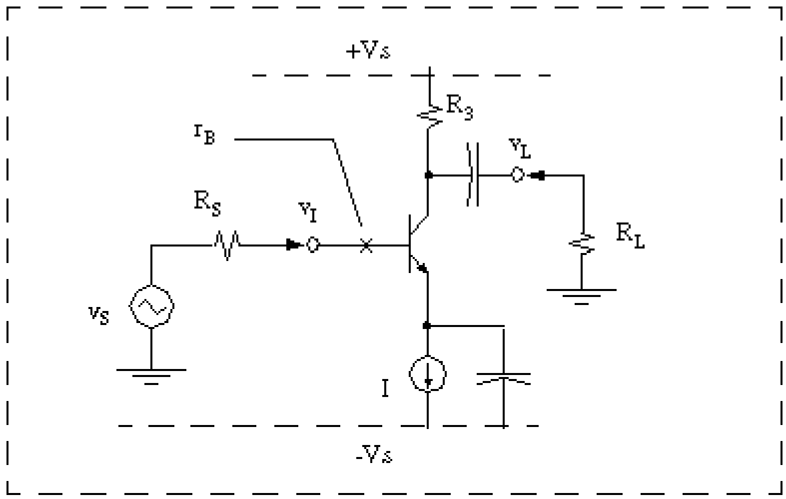

Figure 19.4-3: General-purpose (common-emitter) topology (BJT). Stripped down version. The base-spreading resistance r_B is regarded to be sufficiently small so that it will not appreciably affect the circuit function and so it is shown as an 'x'.

The general purpose (CE) topology is usually the first one in the queue when we first make an acquaintance with transistors. In this case it is also cast in a simplified form for which a relatively simple small-signal equivalent can be used, as represented by figure 19.4-4a.

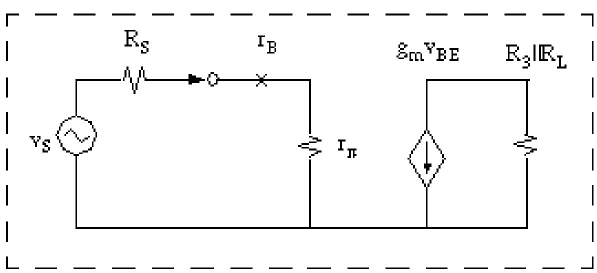

Figure 19.4-4a: Small-signal equivalent of figure 19.2-3. The equivalent load for the transistor = $R_3||R_L = R_L$'. This simplification is put to good use in figure 19.4-2b, which 'exposes' all of the hidden noise sources.

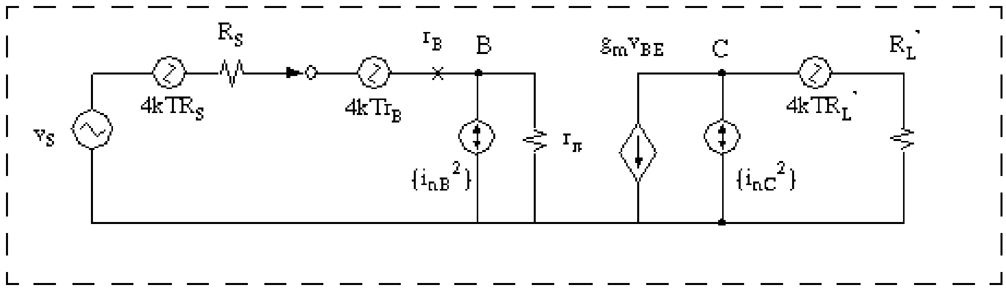

Figure 19.4-4b: Small-signal equivalent of figure 19.2-3 with hidden noise sources exposed.

And had we included more components is developing figure 19.4-4b, as we usually do for this and other circuit topologies, it would have been a very impressive mess. As such, the noise signals that are generated at node *C* are

$$N_{oC} = 4kTR_L^{'} + \{i_{nC}^2\} \times (R_L^{'})^2 \quad = 4kTR_L^{'} + 2qI_C \times (R_L^{'})^2 \qquad (19.4\text{-}10a)$$

and those that are generated at node B are

$$N_{iB} = 4kTr_B + \{i_{nB}^2\} \times (R_S + r_B)^2 \quad \cong 4kTr_B + 2qI_B \times R_S^2 \qquad (19.4\text{-}10b)$$

where it is expected that base-spreading resistance $r_B << R_S$, so we can soften the result a little. Transistor resistance term r_π is a slope and therefore does not generate any noise, but the base-spreading resistance is a small but real resistance.

If we look at the total noise equivalent at the input, then

$$N_{in} = N_{iB} + \frac{N_{iC}}{A_V^2} \quad \cong \left(4kTr_B + N_{iC} \big/ A_V^2\right) + \{i_{nB}^2\} \times R_S^2 \qquad (19.4\text{-}11)$$

for which it should be evident that the equivalent system noise terms and $\{i_n^2\}$and$\{e_n^2\}$as were used by equation (19.4-4) are

$$\{i_n^2\} = \{i_{nB}^2\} \qquad (19.4\text{-}12a)$$

$$\{e_n^2\} = \left(4kTr_B + N_{iC} \big/ A_V^2\right) \quad = \left(4kTr_B + \left[4kTR_L^{'} + 2qI_C \times (R_L^{'})^2\right] \big/ A_V^2\right) \qquad (19.4\text{-}12b)$$

The design merit of this analysis is that many of the noise and transfer terms for the transistor can be resolved in terms of the current $I = I_C$, i.e.

$$\{i_{nC}^2\} = 2qI_C \qquad \{i_{nB}^2\} = 2qI_B = 2q\frac{I_C}{\beta_F} \qquad g_m = \frac{I_C}{V_T}$$

and when applied to equation (19.4-11), combined with (19.4-12), and the fact that $A_V = g_m R_L'$ then

$$N_{in} = N_a \quad = 4kTr_B + \{i_{nB}^2\} \times R_S^2 + \left[4kTR_L^{'} + \{i_{nC}^2\} \times (R_L^{'})^2\right] \big/ g_m^2 \left(R_L^{'}\right)^2$$

$$= 4kTr_B + 2q\frac{I_C}{\beta_F} \times R_S^2 + \left[4kTR_L^{'} + 2qI_C \times (R_L^{'})^2\right] \times \frac{V_T^2}{I_C^2 \left(R_L^{'}\right)^2} \qquad (19.2\text{-}13)$$

if we do a little algebra this simplifies to

$$N_a = 4kTr_B + 2q\left(\frac{I_C}{\beta_F} \times R_S^2 + \frac{V_T^2}{I_C} + 2\frac{kT}{q}\frac{V_T^2}{I_C^2}\frac{1}{R_L^{'}}\right)$$

$$= 4kTr_B + 2q\left(\frac{I_C}{\beta_F} \times R_S^2 + \frac{V_T^2}{I_C} + 2\frac{V_T^2}{I_C^2}\frac{V_T}{R_L^{'}}\right) \cong 4kTr_B + 2q\left(\frac{I_C}{\beta_F} \times R_S^2 + \frac{V_T^2}{I_C}\right)$$

for which the system noise N_a is minimized when

$$\frac{\partial N_a}{\partial I} \cong 2q\left[\frac{R_S^2}{\beta_F} - \frac{V_T^2}{I_C^2}\right] = 0$$

And this corresponds to a noise minimum when [ ] = 0, for which

$$I_C^2 \cong \frac{\beta_F V_T^2}{R_S^2} \qquad (19.2\text{-}14)$$

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EXAMPLE 19.4-2: Design of a low-noise single-transistor (simplified) input stage.

For the circuit shown by figure E19.4-2, determine (a) equivalent input noise terms $\{i_n^2\}$and$\{e_n^2\}$ (b) noise characteristics F and T_e (c) optimum value of R_S (d) optimum value of source current $I = I_C$ if $R_S = 4\text{k}\Omega$ (as shown).

Assume the following defaults for the BJT: $\beta_F = 100$, $r_B = 25\Omega$, and assume V_T (also same as kT/q) = .025V.

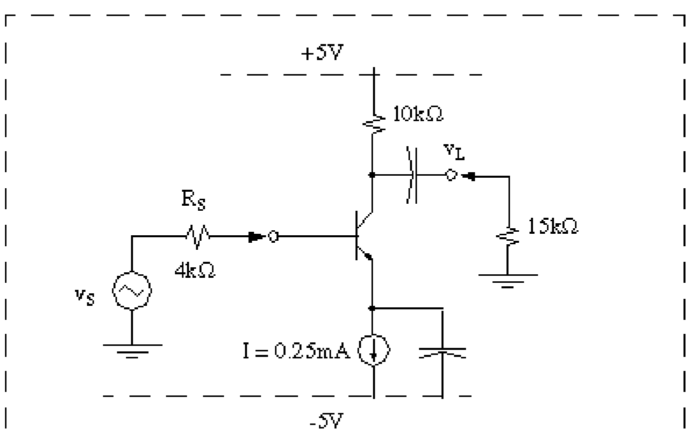

Figure E19.4-2: Input stage using common-emitter (= general purpose) topology.

Analysis: For the current shown,

$$g_m = 40 \times .025mA = 10mA/V$$
$$A_V = \frac{v_C}{v_B} = -10mA/V \times (10k\Omega \,||\, 15k\Omega) = -60V/V$$
$$\{i_{nC}^2\} = 2qI_C = 2 \times 1.6 \times 10^{-19} \times (0.25 \times 10^{-3}) = 0.8 \times 10^{-22}\, A^2/Hz$$
$$\{i_{nB}^2\} = 2qI_B = \frac{2qI_C}{\beta_F} = \frac{1}{100} \times (0.8 \times 10^{-22}) = 0.8 \times 10^{-24}\, A^2/Hz$$
$$4kTr_B = (1.6 \times 10^{-20}) \times 25\Omega = 0.4 \times 10^{-18} V^2/Hz$$

for which

$$N_{oC} = 4kTR_L' + \{i_{nC}^2\} \times (R_L')^2 = (1.6 \times 10^{-20}) \times 6k\Omega + (0.8 \times 10^{-22}) \times (6k\Omega)^2$$
$$= (9.6 \times 10^{-17}) + (28.8 \times 10^{-16}) = 29.76 \times 10^{-16} V^2/Hz$$

and therefore the input equivalent noise terms are

$$\{i_n^2\} = \{i_{nB}^2\} = 0.8 \times 10^{-24}\, A^2/Hz$$

$$\{e_n^2\} = \left(4kTr_B + N_{oC}/A_V^2\right) = 0.4 \times 10^{-18} + 29.76 \times 10^{-16}/(60)^2 = 1.24 \times 10^{-18} V^2/Hz$$

*Since the exercise involves something of a calculation grind, take note of the systematic process that needs to be followed in order to keep magnitudes straight. Also take note of the fact that the values for $\{i_n^2\}$and$\{e_n^2\}$ are similar in magnitude to those for $\{i_n\}$and $\{e_n\}$ extracted in Exercise E19.4-1.

In like manner, $4kTR_S =$ $(1.6 \times 10^{-20}) \times (4 \times 10^3) = 6.4 \times 10^{-17} V^2/Hz$

and $$F = 1 + \frac{1.24 \times 10^{-18} + (4 \times 10^3)^2 \times (0.8 \times 10^{-24})}{6.4 \times 10^{-17}} = 1 + \frac{0.124 + 1.28}{6.4} = \boxed{1.22}$$

for which T_e = (1.22 -1) x 300 = $\boxed{66K}$

(c) The optimum value of R_S would have been

$$R_S(optimum) = \sqrt{\frac{1.24 \times 10^{-18}}{0.8 \times 10^{-24}}} = \boxed{1.25k\Omega}$$

(d) the optimum value of I_C would have been

$$I \cong \sqrt{\beta_F}\,\frac{V_T}{R_S} = \sqrt{100} \times \frac{.025}{4.0k} = \boxed{.0625mA}$$

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19.5 NOISE NOMENCLATURE AND ANALYSIS USING SPICE

It should be evident from exercise 19.4-2 that noise analysis can entail an overload of computational overhead. Each resistor and transistor contributes to the noise calculation as either a noise current or a noise voltage generator. The circuit simulation environment can handle this overhead without too much complaint. It does so via the noise-spectral density (NSD), and the user can assess full-spectrum effects using post-processor utilities.

In the SPICE simulation environment, each noise source is assessed one-by-one as a source embedded within the circuit topology, and gives its result in terms of the individual and collective effect at the output node. Note that the output point must therefore be identified. Results in each instance are in terms of V^2/Hz, not unlike that entertained in section 19.4.

The specific mathematics of the noise contributions will be associated with the topology. For example the SPICE nomenclature NTOT(Rs) identifies the noise contribution from the resistance R_S at the output, for which

$$NTOT(Rs) = (4k_B T R_S) \times (input\_Vdiv)^2 \times (v_L / v_I)^2 \qquad (19.5\text{-}1)$$

where the input voltage divider (*input_Vdiv*) depends on R_{in} and R_S, and the transfer gain v_L/v_I depends on the circuit topology.

For a shot noise source such as that of the form of equation (19.4.7a) (= base current noise) SPICE uses the nomenclature NSIB(Q1), for which Q1 is the affiliated transistor. The specific mathematics for the term NSIB(Q1) is

$$NSIB(Q1) = (i_{nB}^2 \times R_{BS}^2) \times (base\_Vdiv)^2 \times (v_L / v_B)^2 \qquad (19.5\text{-}2)$$

where the *base_Vdiv* is the voltage divider that a source sitting at the base node will see, and v_L/v_B is the transfer gain to the output node from the base node.

This would be a fair amount of overhead grind if done by hand, but of course SPICE can do all of this quite handily and with much less trauma.

In any case SPICE does all of this in the user’s behalf. It provides terms, with names such as NTOT(Rs), NSIB(Q1), NRB(Q1), each of which are individual contributions to the NSD as read at the output. If you don’t know exactly what each may be, you have to make some guesses according to the nomenclature.

The total contribution at the output is given by NTOT(ONOISE). This is a noise density , and therefore it has an amplitude that follows the pass band of the circuit. The total output noise is the sum of all of the noise contributions over the entire spectrum, and can be accomplished by the pSPICE summing (same as integral) function *S( )* It sums up all of the contributions for the entire spectrum by means of

$$e_n^2(out) = \int_0^\infty NTOT(ONOISE)\,df \qquad (19.5\text{-}3)$$

which is accomplished by the SPICE function S(NTOT(ONOISE)). Signal/noise ratio is then obtained by

$$\frac{S}{N} = \frac{v_L^2}{e_n^2(out)} = \frac{Max(Vout) * Max(Vout)}{Max(S(NTOT(ONOISE)))} \qquad (19.5\text{-}4)$$

Noise factor (relative contribution due to the amplifier) can be obtained by

$$F = \frac{Max(NTOT(ONOISE))}{Max(NTOT(RS))} \qquad (19.5\text{-}5)$$

for which NTOT(RS) is the noise contribution at the output due to the input source resistance R_S, whereas NTOT(ONOISE) is due to everything, including R_S.

Using these postprocessor functions, and the parameter declaration and variation option, noise analysis characteristics such as those of (19.5-4) and (19.5-5) can be evaluated for the effect of circuit design choices such as R_S and I_C.

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EXAMPLE 19.5-1: Determine R_S*(opt)* and the associated noise factor *F* for the CE circuit topology of figure E19.5-1 using pSPICE.

Analysis: Execute a frequency sweep from 50Hz to 50MHz with the noise enabled. Using the PARAMETER call and the **Analysis>Setup>Parametric** menu, vary the source resistance R_S from 1k to 10k, call up the goal function menu for the post processer, and plot noise factor *F* (given by equation (19.5-5)) vs R_S. Find the value of R_S for which *F* is a minimum.

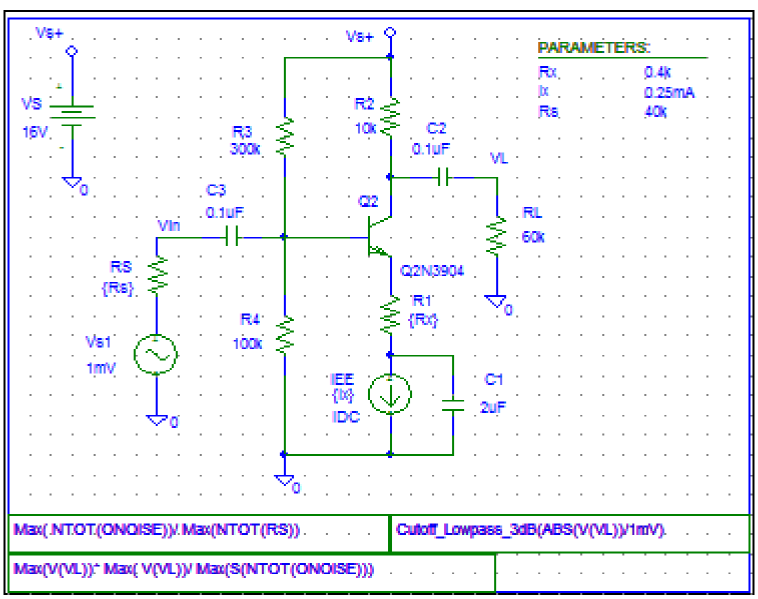

Figure E19.5-1a: Single-rail common emitter configuration. Note: It is good (and necessary) form to include the goal function macro as a text line on the schematic.

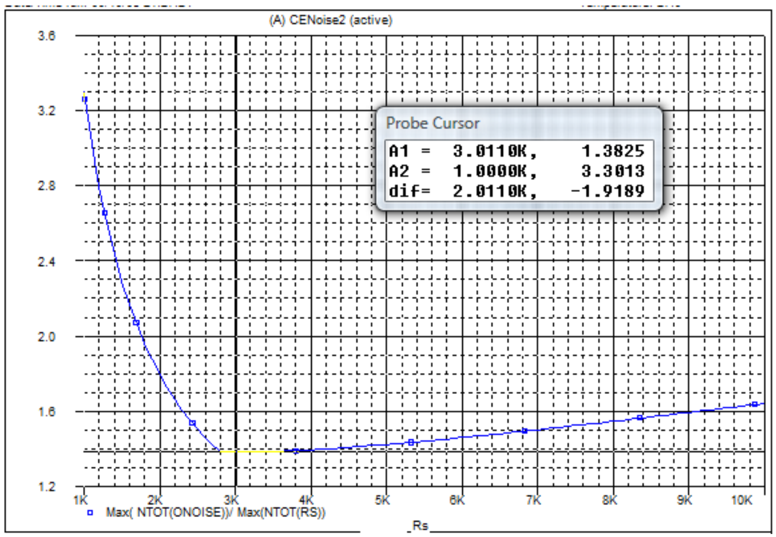

Figure E19.5-1b: **(Results of SPICE analysis)** The trace and cursor show that noise factor F is a minimum at R_S = 3kΩ, and has value F = 1.3825, (same as T_e = 115°K).

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19.6 DYNAMIC RANGE AND CUBIC DISTORTION

Distortion is a consequence of amplitude, in which the small-signal linearity we associate with the amplifier has been exceeded. Generally speaking, most distortion manifests itself as a "flattening" of the waveform as result of the signal approaching the compliance limits. It is also a result of the linear approximation associated with the active devices (transistors) being exceeded, in which case it will manifest itself as higher-order algebraic and Fourier analysis terms.

In the analysis of the power gain, quadratic terms are usually eliminated by means of a differential stage (such as an opamp). That means that the next higher-order terms will be cubic. This distortion is represented by figure 19.6-1, and can be regarded as if it were a mixing of signals, as shown. Consequently it is also often called *intermodulation distortion*.

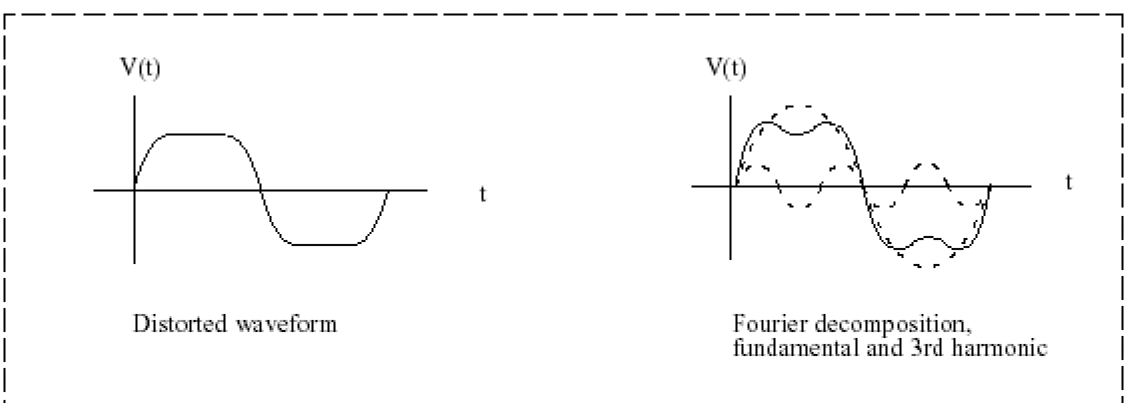

Figure 19.6-1: Waveforms and cubic distortion

Distortion power for cubic distortion relates to the input power level as

$$P_d = kP_i^3 \qquad (19.6\text{-}1)$$

A plot of the relative output levels is represented by figure 19.6-2.

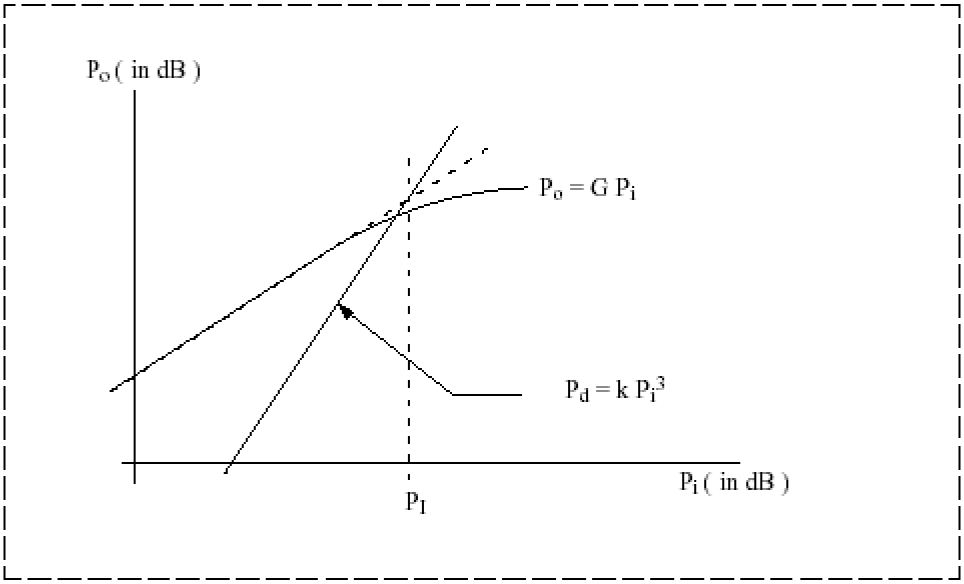

Figure 19.6-2: Output power P_O vs cubic distortion power as represented by equation (19.6-1)

At some input level of power, $P_i = P_I$, the power associated with cubic distortion begins to dominate the output power.

If we compare P_d vs P_i by means of a ratio

$$\frac{P_d}{P_O} = \frac{kP_i^3}{GP_i} = (K_i P_i)^2 \qquad (19.6\text{-}2)$$

where G = power gain of the amplifier. And since $P_d / P_O = 1$ when $P_i = P_I$ then $K_i = 1/P_I$ and

$$\frac{P_d}{P_O} = \left(\frac{P_i}{P_I}\right)^2$$

If we treat the distortion power as if it were due to an input level P_{di} then

$$\frac{P_d}{P_O} = \frac{GP_{di}}{GP_i} = \frac{P_{di}}{P_i} = \left(\frac{P_i}{P_I}\right)^2 \qquad (19.6\text{-}3)$$

Now, if we treat P_{di} as if it were a power limit to the input at which the signal quality falls below useful levels, this correlates exactly to that for the noise floor, *Nf*. And then equation (19.6-3) gives

$$\frac{Nf}{P_i} = \left(\frac{P_i}{P_I}\right)^2$$

which corresponds to limit on input power P_i of

$$P_i = \left(P_I^2 Nf\right)^{1/3} \tag{19.6-4}$$

The dynamic range was identified in section 19.1 as

$$DR = 10\log\left(\frac{\text{Pi = distortion limit power}}{\text{noise limit power = Nf}}\right)$$

therefore

$$DR = 10\log\left(\frac{\left(P_I^2 Nf\right)^{1/3}}{Nf}\right) = 10\log\left(\frac{P_I}{Nf}\right)^{2/3}$$

so that the dynamic range for cubic distortion reduces to the form

$$DR = \frac{2}{3}\left(10\log P_I - 10\log Nf\right) \tag{19.6-5}$$

And for most cases of signal conditioning for which we are assessing the limits if a system, this is the definition of dynamic range that will be assumed. Note that equation (19.6-5) is oriented toward P_I and Nf being expressed in units of either dBW or dBm.

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EXAMPLE 19.6-1: Assume that the system identified by example 19.3-2 has an intercept point P_I = -10dBW. (a) what is the dynamic range of the system (b) what is the DR of the system if a linear noiseless preamplifier of gain 10dB is inserted before the first stage?

SOLUTION: The result of example 19.2-2 was that noise floor Nf = -153 dBW

(a) From equation (19.6-5) we then get a DR of

$$DR = \frac{2}{3}\left(-10dBW - (-153dBW)\right) = -95.3dB$$

(b) If we add a preamplifier of power gain G_1 = 10dB, then distortion will occur for an even lower power input level, namely P_I'=-20dBW. Recalculating the noise factor, and assuming value of F_2 = 6.61, as identified by example 19.2-2 we get

$$F' = F_1 + \frac{F_2 - 1}{G_1} = 1 + \frac{6.31 - 1}{10} = 1.53$$

which corresponds to 1.85dB, for which the noise floor will then become Nf = -159.4dBW and the dynamic range will be

$$DR = \frac{2}{3}\left(-20dBW - (-159.4dBW)\right) = -92.9dB$$

Take note that the added amplifier serves to reduce the noise floor, but concurrently will also decrease the DR, courtesy of the effects on the distortion, as reflected by the change in the intercept point P_I .
****